University of California, Berkeley
Department of Mathematics
10^{th} August, 2012, 8:00-10:00 am
MATH 53 - Final Exam

Last Name:____

First Name:____

Student Number:____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

Problem	Score
1	/12
2	/10
3	/14
4	/16
5	/16
6	/12
Total	/80

1. Let S be the oriented surface given by the vector-valued function

$$\vec{r}(u,v) = \langle v^2, -uv, u^2 \rangle, \quad u \in [0,3],\, v \in [-3,3].$$

[6] (a) Find the equation of tangent plane to S at the point $(4,-2,1)$.

[4] (b) Set up, but do not evaluate, the integral that will compute the surface area of S.

[2] (c) At what (if any) points is the tangent plane to S horizontal?

2. Let D be the region in the first quadrant bounded by the circles $x^2+y^2=4$ and $x^2+y^2=9$, and the hyperbolas $x^2-y^2=1$ and $x^2-y^2=4$.

[3] (a) Sketch the region D.

[7] (b) Evaluate the integral $\iint_D xy\,dA$.

Hint: the boundary curves for D given above should suggest how to define the inverse transformation T^{-1} that gives u and v in terms of x and y. It won't be necessary to solve for x and y in terms of u and v in order to compute the Jacobian (although you can). Instead, use the relationship

$$J_T(u,v) = \frac{1}{J_{T^{-1}}(x(u,v),y(u,v))}.$$

3. Let E be the solid region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{2 - x^2 - y^2}$. (You may want to sketch the region.)

   [7] (a) Using **cylindrical coordinates**, find the mass of the solid occupying the region E if its mass density is given by $\delta(x, y, z) = \lambda z$, where λ is a positive constant.

   [7] (b) Using **spherical coordinates**, find the volume of the solid bounded by E.

4. Consider the vector field $\vec{F}(x,y) = (3x^2 + 2y^2)\hat{\imath} + (4xy + 6y^2)\hat{\jmath}$.

[2] (a) Show that $\vec{F}(x,y)$ is conservative.

[5] (b) Find a function $f(x,y)$ such that $\nabla f(x,y) = \vec{F}(x,y)$.

(c) If C is given by the parabolic arc $x = 2y^2$ from $(0, 0)$ to $(2, 1)$, followed by the line segment from $(2, 1)$ to $(-1, 3)$, compute the line integral $\int_C \vec{F} \cdot d\vec{r}$:

[7] (i) Directly.

[2] (ii) Using the Fundamental Theorem of Calculus for line integrals.

5. Let S be the surface given by $z = 4 - x^2 - y^2$, for $0 \leq z \leq 3$, oriented with outward-pointing normal vector field, and let $\vec{F} = \langle yz, -xz, z^3 \rangle$.

[3] (a) Sketch the surface, and indicate the direction of its positively-oriented boundary curve(s) C.

[2] (b) Compute $\nabla \times \vec{F}$.

[4] (c) Explain why Green's Theorem is a special case of Stokes' Theorem.

[7] (d) Compute $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$.

(Hint: use Stokes' Theorem twice.)

[12] 6. Verify that the Divergence Theorem is true for the vector field $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and the region E given by the solid ball $x^2 + y^2 + z^2 \leq 9$.

 Total Marks: 80