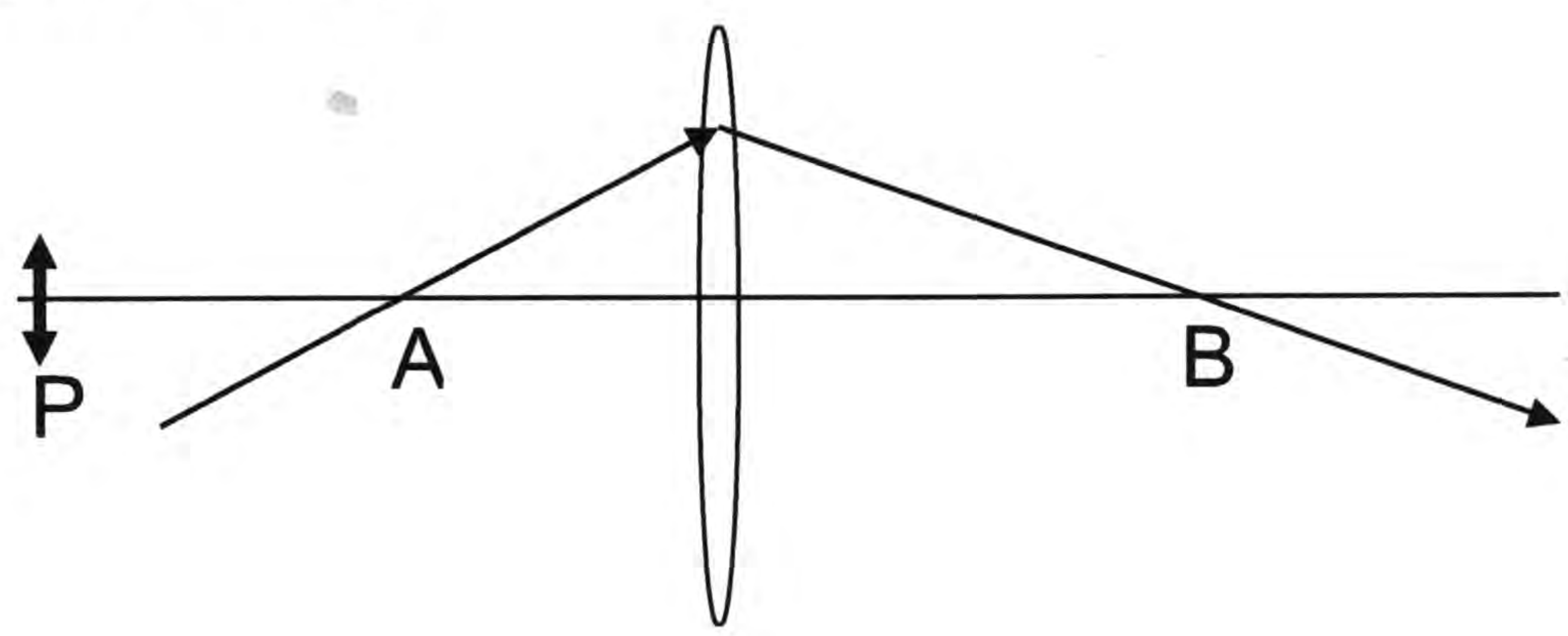
University of California, Berkeley Physics
H7C Spring 2012
MidTerm Exam II on Optics
Maximum score: 100 points

1. For a thin lens, we know one of the light ray will be refracted as shown in Fig. 1. Here point A is at 30 cm to the left of the lens and point B is at 50 cm to the right of the lens.

a) (10 points) Determine the focal length of the lens.

b) (10 points) If an erected object P of height 10cm is put at position 60 cm to the left of the lens, what will be the position and the height of its image?

c) (10 points) If we put an opaque screen to block the lower half of the lens, describe what will happen to the image.

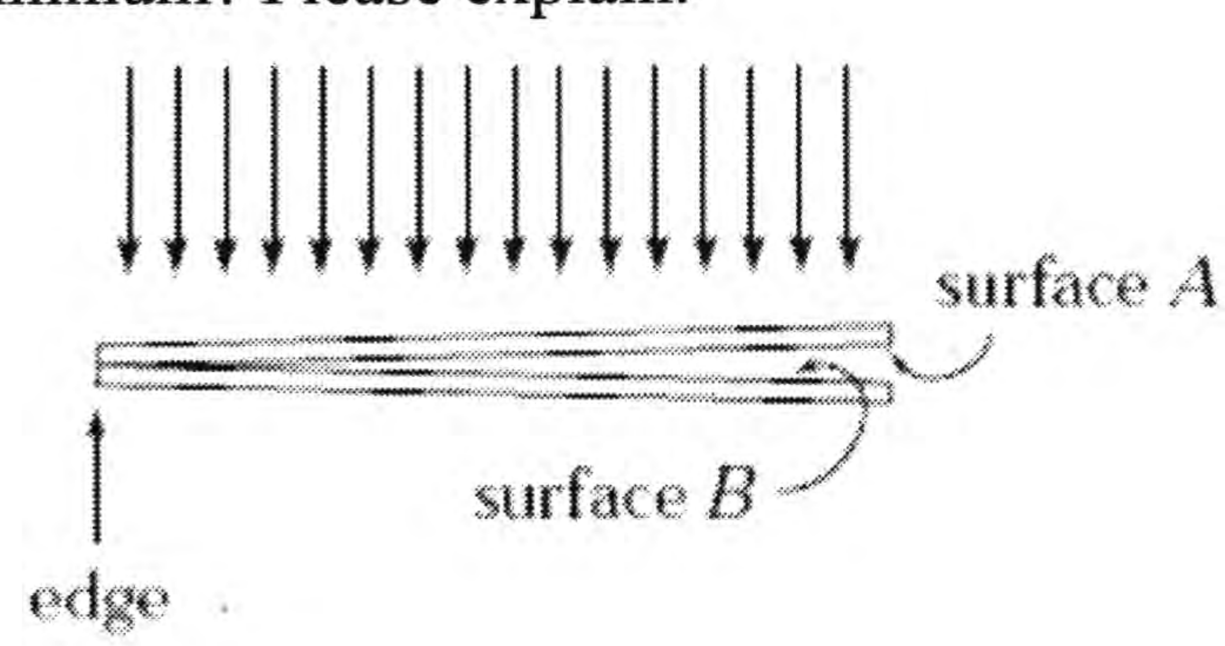


2. Monochromatic light with a wavelength of 500 nm shines on a pair of identical glass microscope slides that form a very narrow wedge with an angle of 0.001 radiant. The top surface of the upper slide and the bottom surface of the lower slide have special coatings on them so that they reflect no light. The inner two surfaces (A and B) have nonzero reflectivities.

From the top of the glass wedge, we will observe interference of the reflected light.

(a) (20 points) What is the lateral distance between the interference maximum?

(b) (10 points) At the edge of the wedge, will there be an interference maximum or minimum? Please explain.

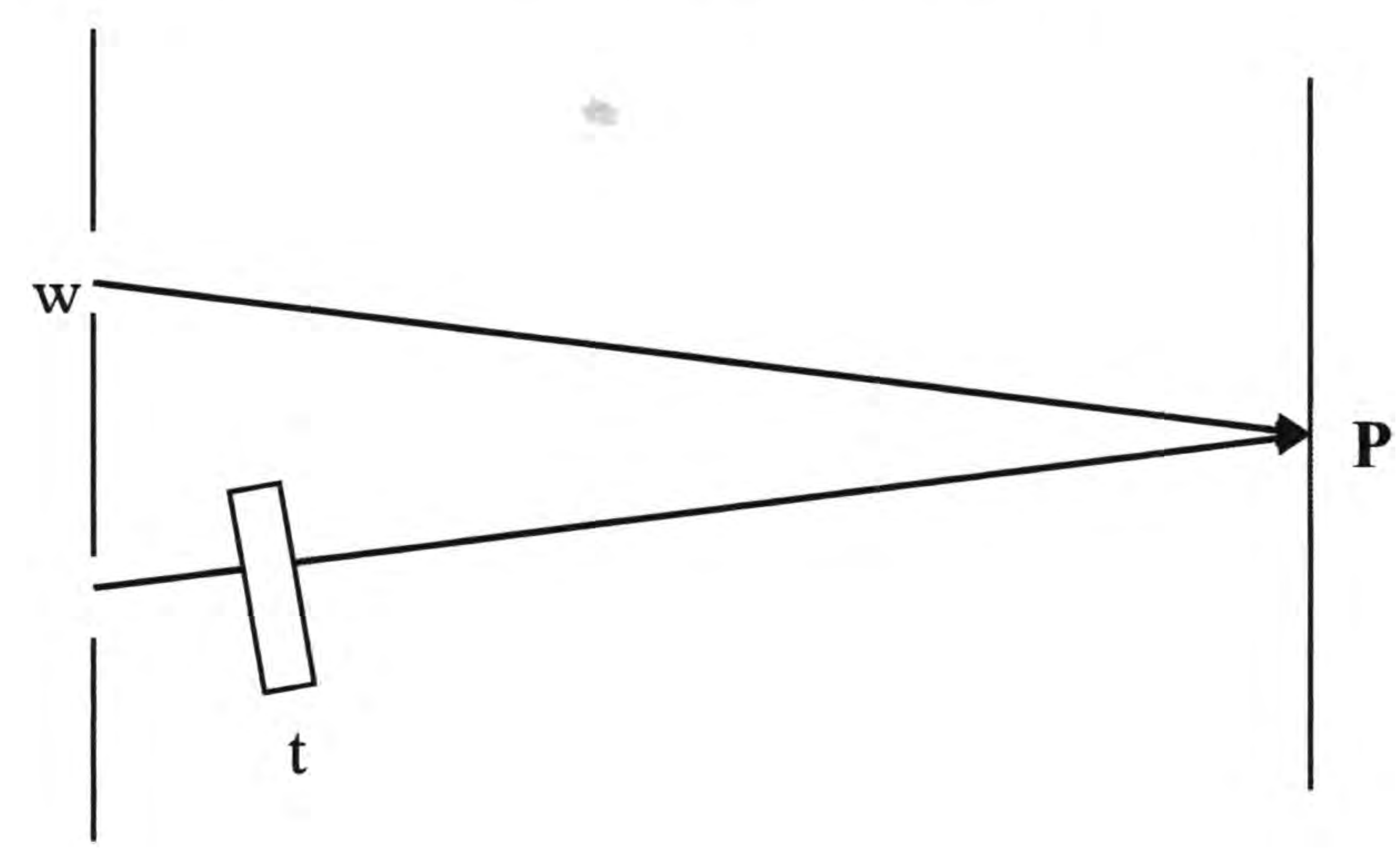


3. The figure below shows a double-slit experiment in which coherent monochromatic light of wavelength λ from a distant source is incident upon the two slits, each of width w ($w >> \lambda$), and the interference pattern is viewed on a distant screen. A thin peach of glass of thickness t, index of refraction n, is placed between one of the slits and the screen perpendicular to the light path. Assume that the glass does not absorb or reflect any light. If the intensity of light at the central point P for $t=0$ is given by I_0,

(a) (20 points) What is the intensity at point P as a function of thickness t?

(b) (10 points) For what values of t is the intensity at P a minimum?

(c) (10 points) Suppose the width w of one of the slits is now increased to 3w, the other width remains at w. What is the intensity at point P as a function of t?



H7C, S12, MT2 Solutions

1a) You can think of this ray as ~~an~~ that produced by an object of zero height at A with an image of zero height at B. Then $S_o = 30$ cm, $S_i = 50$ cm and

$$\frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_i} \rightarrow f = \frac{S_o S_i}{S_o + S_i} = \frac{30 \cdot 50}{80} = \frac{150}{8} = \boxed{18.75 \text{ cm}}$$

b) Now $S_o = 60$ cm. Therefore

- $$\frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} \rightarrow S_i = \frac{f S_o}{S_o - f} = \frac{(18.75)(60)}{41.25} = \frac{300}{11} = \boxed{27.27 \text{ cm}}$$

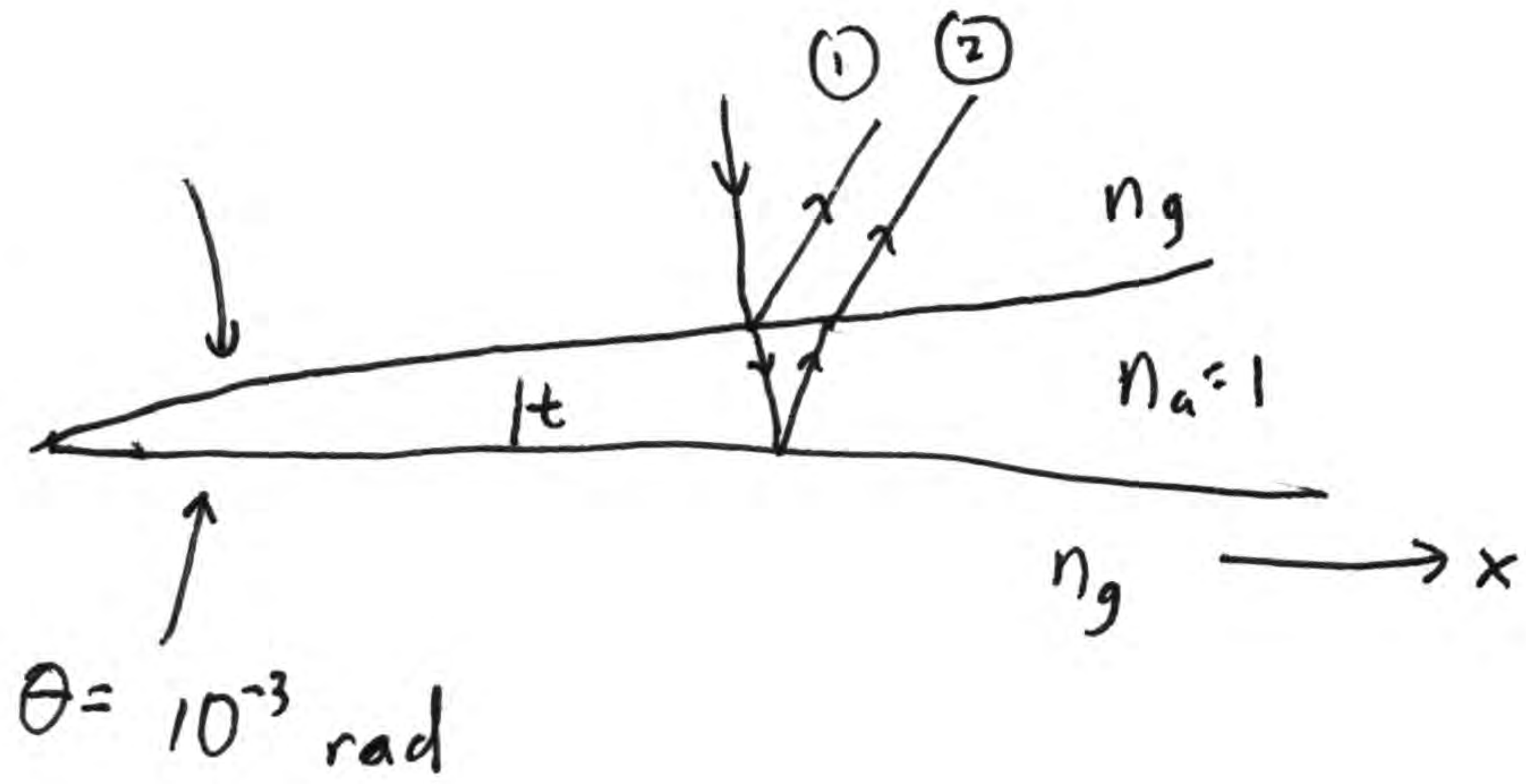
- $$\frac{y_i}{y_o} = \frac{-S_i}{S_o} \rightarrow y_i = 10 \cdot \frac{-300/11}{60} = -\frac{50}{11} = -4.55 \text{ cm}$$

→ image is 27.27 cm to the right of the lens, is inverted, and has height 4.55 cm

c) Only half of the rays going from the object to the image get through the lens, so the image is half as bright (but the entire image is still visible).

2a) Interference maxima occur where the difference in phase $\Delta\varphi$ between two beams is a multiple of 2π:

$$\Delta\varphi = 2n\pi, \quad n = 0, 1, 2, \cdots$$



for beam ①, there is no phase picked up from reflection, because $n_g > n_a$.

for beam ②, there is an extra phase of π due to reflection ($n_a < n_g$) and an extra phase of $2tk = \frac{4\pi t}{\lambda}$ due to the extra distance the light travels.

Hence, $\Delta\varphi = \pi + \frac{4\pi t}{\lambda} \rightarrow 2n\pi = \pi + \frac{4\pi}{\lambda}\theta x$

$$\rightarrow x = \frac{\lambda}{4\theta}(2n-1)$$

$$\Delta x = \frac{\lambda}{4\theta}(2) = \frac{500 \times 10^{-9}}{2 \times 10^{-3}} = 250 \times 10^{-6} = \boxed{0.25\,mm}$$

b) At the edge, $t = 0$ so $\Delta\varphi = \pi \rightarrow$ interference minimum

3a) The fact that $\lambda \ll w$ means that we can ignore diffraction effects, that is, we can think of the light coming from each slit to P as a single wave of some unknown Amplitude.

In general, the intensity of two interfering waves is given by

$$I(\Delta\varphi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi$$

where $\Delta\varphi$ is the phase shift between the rays.

For this problem, $\Delta\varphi = t\, n_{glass}\frac{2\pi}{\lambda} - t\, n_{air}\frac{2\pi}{\lambda} = \frac{2t\pi}{\lambda}(n-1)$

The slits are identical, so $I_1 = I_2$. We also know that $I(\Delta\varphi = 0) = I_0$. Therefore,

$$I_0 = 4I_1 \rightarrow \boxed{I(t) = \frac{I_0}{2}\left(1 + \cos\left[\frac{2\pi t}{\lambda}(n-1)\right]\right)}$$

$\hookrightarrow I_1 = I_2 = \frac{I_0}{4}$

b) Minima occur when $\cos\left[\frac{2\pi t}{\lambda}(n-1)\right] = -1$

$$\rightarrow \frac{2\pi t}{\lambda}(n-1) = (2m+1)\pi \quad \text{for integer } m$$

$$\rightarrow \boxed{t = \frac{\lambda}{n-1}\left(m + \frac{1}{2}\right)}$$

c) If the width of a slit increases by 3, This effectively triples the amplitude of the total electric field coming from that slit, and hence the intensity goes up by a factor of $3^2 = 9$.

(ie, $I_1 = \frac{I_0}{4}$; $I_2 = \frac{9I_0}{4}$)

Otherwise, the calculation proceeds as in part a):

$$I(\Delta\varphi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\varphi)$$

$$= \frac{I_0}{4} + \frac{9I_0}{4} + 2\,\frac{3}{4} I_0 \cos(\Delta\varphi)$$

$$= \frac{5I_0}{2} + \frac{3I_0}{2}\cos(\Delta\varphi)$$

$$\rightarrow \boxed{I(t) = \frac{I_0}{2}\left(5 + 3\cos\left[\frac{2\pi t}{\lambda}(n-1)\right]\right)}$$