**University of California, Berkeley Physics**

**H7C 2012 MidTerm Exam I**
***Relativity, Electromagnetic Waves***

Maximum score: 100 points

1. (35 points) Suppose you direct a laser beam with frequency $f_0$ at an atom moving towards you with a velocity u.

(a) (10 points) What is the light frequency felt by the atom in its rest frame.

(b) (10 points) The atom will be driven by the laser beam and re-radiate. What is the frequency of the light radiated by the atom in its reference frame?

(c) (5 points) If you observe this atom radiation, what light frequency will you see? What is the corresponding light wavelength?

(d) (10 points) Suppose you now direct the laser beam towards a mirror moving towards you with a velocity u. What will be the light frequency that you observe? Briefly explain why.

2. (35 points) A fast moving particle of rest mass $M_0$ decays into two particles of equal rest mass $m_0$. In the laboratory, one of the resulting particles is at rest, while the other has a kinetic energy of $3m_0c^2$. Obtain an expression for the rest mass $M_0$ in terms of $m_0$.

3. (30 points) Using a block of transparent, unknown materials, it is found that a beam of light inside the material is totally internally reflected at the block-air interface at a minimum angle of 42.6 degree.

(1) (15 points) What is the index of refraction of this material?
(2) (15 points) If another beam of light inside this material have 100% transmission into the air. What is the incident angle of this beam of light at the block-air interface? And what is the light beam polarization?

1a) let's assume that $p_{light} << p_{atom}$ so that the laser more or less doesn't change the speed of the atom. Then we can define ~~the~~ a rest frame for the atom.

The frequency as seen by the atom will be doppler shifted: $\boxed{f_1 = f_0\sqrt{\frac{1+\beta}{1-\beta}}}$

b) So, "driven and re-radiated" means that the photon is absorbed by the atom and then emitted again. Also, we have to know that $E=hf$ for the photon (the Energy equals the frequency times Planck's constant). So, assuming conservation of energy in the atom's frame, ~~the~~

$$\boxed{f_{emitted} = f_1}$$

c) Now we jump back into the lab frame. Again, asume that the atom didn't change speed. Also assume that the atom radiates light back towards the laser. Then the frequency seen by you is

$$f_2 = \sqrt{\frac{1+\beta}{1-\beta}}\, f_{emitted} = \boxed{\left(\frac{1+\beta}{1-\beta}\right) f_0} \quad ; \quad \lambda_2 = \frac{c}{f_2} = \boxed{\frac{c}{f_0}\left(\frac{1-\beta}{1+\beta}\right)}$$

d) Essentially, the mirror . acts like a bunch of these atoms,

so $\boxed{f = \left(\frac{1+\beta}{1-\beta}\right) f_0}$

2) In the Lab frame:


$$E = \gamma M_0 c^2 \qquad E_1 = m_0 c^2 \qquad E_2 = \gamma' m_0 c^2 = K + E_{rest}$$

$$cp = \gamma \beta M_0 c^2 \qquad cp_1 = 0 \qquad cp_2 = \gamma' \beta' m_0 c^2 = 3 m_0 c^2 + m_0 c^2$$

$$= 4 m_0 c^2$$

Cons. of Energy: $\gamma M_0 c^2 = m_0 c^2 + 4 m_0 c^2 = 5 m_0 c^2 \cdots (1)$

cons of momentum: $\gamma \beta M_0 c^2 = \gamma' \beta' m_0 c^2 \cdots\cdots (2)$

invariant for 2: $E_2^2 - (cp_2)^2 = (m_0 c^2)^2$

$$(4 m_0 c^2)^2 - (\gamma' \beta' m_0 c^2)^2 = (m_0 c^2)^2$$

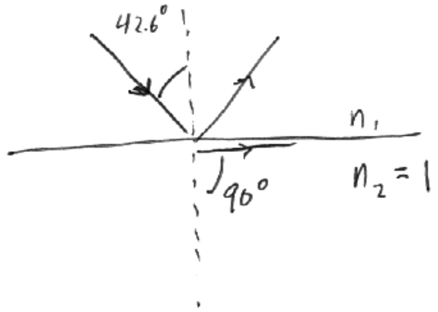
$$16 - (\gamma' \beta')^2 = 1 \Rightarrow \gamma' \beta' = \sqrt{15} \cdots (3)$$

So now, combining (1), (2), and (3), we get:

$$\left. \begin{array}{l} \gamma = 5 \frac{m_0}{M_0} \\ \gamma \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{15} \frac{m_0}{M_0} \end{array} \right\} \to 5 \frac{m_0}{M_0} \sqrt{1 - \left(\frac{M_0}{5 m_0}\right)^2} = \sqrt{15} \frac{m_0}{M_0}$$

$$\to \boxed{M_0 = \sqrt{10}\, m_0}$$

3a)

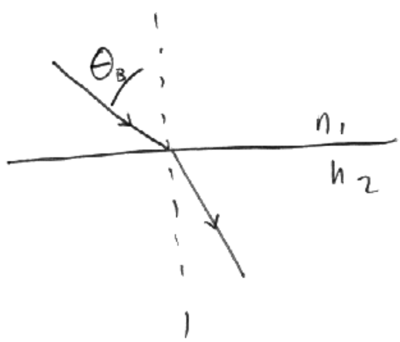



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 = n_2 \frac{\sin\theta_2}{\sin\theta_1} = 1 \frac{\sin 90^\circ}{\sin 42.6^\circ}$$

$$= \boxed{1.48}$$

b)




No reflection happens at Brewster's angle for light polarized **parallel** with the plane of incidence.

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1}{1.48}\right)$$

$$= \boxed{34.1^\circ}$$