1.(5 points) *Consider the series*

$$\sum_{n=1}^{\infty} \frac{5^n}{n!\, 2^n}.$$

ⓐ *Show that the series converges.*

ⓑ *What does it converge to?*

2. (7 points) *Consider the function*

$$f(x) = x\cos(\frac{x^2}{2}),$$

ⓐ *Find the Taylor series of $f(x)$ around $x = 0$, and its radius of convergence.*

ⓑ *Find the limit*

$$\lim_{x\to 0} \frac{x\cos(x^2/2) - x + x^5/4}{x^5}$$

ⓒ *Find the Taylor series for*

$$\int f(x)dx,$$

around $x = 0$, and its radius of convergence.

3. (6 points) *Consider the series*

$$\sum_{n=0}^{\infty} ne^{-nx}$$

ⓐ *Find the values of x for which the series converges.*

ⓑ *What function of x does the series converge to, in its interval of convergence? (Hint: consider a change of variables.)*

4. (6 points) *Consider the series*

$$\sum_{n=1}^{\infty} \frac{1}{n^5}$$

ⓐ *Show that one can use integral test to study this series.*

ⓑ *Prove, using the integral test, that the series converges.*

ⓒ *Estimate the error in approximating the series by its first term. Use this to give an estimate for the value of the sum.*

5.(6 points) *Consider the series*

$$\sum_{n=2}^{\infty} (\ln n)^p \frac{1}{n} (-1)^n.$$

For which values of $p \in \mathbb{R}$ the above series

ⓐ *converges absolutely,*

ⓑ *converges conditionally,*

ⓒ *diverges.*