Physics 137A, Spring 2004 , Section 1 (Hardtke), Midterm I

You are allowed one 8 1/2 by 11 page (both sides) with notes. Calculators allowed. Put your answers on separate sheets or an exam book. Write your name on each page.
There are 4 questions and a page of integrals and constants you might need.

1. The largest objects for which quantum interference has been observed are Carbon Fullerenes ("buckyballs"). These are C_{60} molecules containing 60 carbon atoms. Carbon contains 6 protons and 6 neutrons.

   (a) What is the De Broglie wavelength for buckyballs with kinetic energy $E = 0.1$ eV? Calculate your result to one significant figure. The two-slit interference pattern for these buckyballs is shown in the figure. (10 points)

   (b) For the data shown, the buckyballs have a temperature of approximately 900K. In a paper published last month, it was shown that if the buckyballs are heated to a temperature of 3000K, the two-slit interference pattern disappears. Provide a simple *qualitative* explanation for this observation. [Hint: A buckyball can be treated as a blackbody radiator.] (5 points)

2. You are given an arbitrary potential $V(x)$ and the corresponding orthonormal bound-state solutions to the time-independent Schrödinger Equation, $\psi_n(x)$ and E_n. At time t=0, your system is in the state,

$$\Psi(x, 0) = A(\psi_1(x) + \psi_2(x) + \psi_4(x)).$$

   (a) Find A to normalize the wave function. (5 points)

   (b) What is the wave function at time $t > 0$? (10 points)

   (c) What is the expectation value of the energy at time $t > 0$? (10 points)

3. At time $t = 0$, a free non-relativistic electron is moving in one dimension along the x-axis in a state described by the wave function:

$$\Psi(x, 0) = \begin{cases} 0 & \text{for } x < -a \\ -A & \text{for } -a \leq x < 0 \\ A & \text{for } 0 \leq x < a \\ 0 & \text{for } x \geq a \end{cases}$$

(a) Find A to normalize the wave function. (5 points)

(b) Determine the momentum-space wave function ($\Phi(p, t)$) at t=0. Simplify the expression as much as possible. You should end up with 2 terms.(15 points)

(c) Calculate $\langle x \rangle$, $\langle x^2 \rangle$, and $\langle p \rangle$ at time t=0. (5 points each)

4. A one-dimensional infinite square well has the potential,

$$V(x) = \begin{cases} \infty & \text{for } x < -a/2 \\ 0 & \text{for } -a/2 \leq x \leq a/2 \\ \infty & \text{for } x > a/2 \end{cases}$$

The energy levels for this potential are of course identical to those we found in class (where the well extended from 0 to a). The wave functions are,

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} sin(\frac{n\pi x}{a}) & N = 2, 4, 6, \ldots \\ \sqrt{\frac{2}{a}} cos(\frac{n\pi x}{a}) & N = 1, 3, 5, \ldots \end{cases}$$

Assume that a particle is the ground state of this symmetric infinite square well,

$$\psi(x) = \sqrt{\frac{2}{a}} cos(\frac{\pi x}{a}).$$

At some instant, the walls are moved very suddenly to $x = \pm b/2$ with $b > a$.

(a) What is the probability that the particle will be found in the ground state of the new potential? Do not try to simplify your answer, although you should do any necessary integrals.(15 points)

(b) What is the probability that the particle will be found in the first excited state of the new potential? (5 points)

(c) What is the probability that the particle will be found in any odd (anti-symmetric) state of the new potential? (5 points)