BIOE 110: Biomedical Physiology for Engineers
Spring 2013
Midterm I Solutions Key

QUESTION 1

Consider a chamber (at 25°C) consisting of two 1L solutions of mannose (MW 180 g/mol) separated by a semipermeable membrane. Assume that mannose has a reflection coefficient across the membrane of 0.75 and is 100% soluble under these conditions. **(20 total)**

A. If osmotic pressure gradient across membrane is 167 kPa, and the more dilute of the two solutions has a concentration of 10 mM, how much mannose (in g) must have been dissolved in the other solution? **(10)**

Solution: Remember that the equation for osmotic pressure is given by $\pi = gC\sigma RT$. The osmotic pressure, π_1, of the more dilute chamber (Chamber 1) is given by:

$$\pi_1 = gC\sigma RT$$

$$\pi_1 = (1)\left(10\frac{\text{mmol}}{\text{L}}\right)(0.75)\left(0.082\frac{\text{L-atm}}{\text{mol-K}}\right)(298\text{K})\left(\frac{1\text{ mol}}{1000\text{ mmol}}\right)$$

$$\pi_1 = 0.183\text{ atm}$$

Since 101.3 kPa is equal to 1 atm, the osmotic pressure is approximately 18.5 kPa. We also know the osmotic pressure gradient across the membrane, 167 kPa. Hence:

$$\Delta\pi = \pi_2 - \pi_1$$

$$167\text{ kPa} = \pi_2 - 18.5\text{ kPa}$$

$$\pi_2 = 185.4\text{ kPa}$$

Now that we know the osmotic pressure of the second, more concentrated chamber, we can solve for the concentration of mannose in this chamber as follows:

$$\pi_2 = gC\sigma RT$$

$$185.4\text{ kPa}\left(\frac{1\text{ atm}}{101.325\text{ kPa}}\right) = (1)C(0.75)\left(0.082\frac{\text{L-atm}}{\text{mol-K}}\right)(298\text{K})\left(\frac{1\text{ mol}}{1000\text{ mmol}}\right)$$

$$C = \frac{185.4\text{ kPa}\left(\frac{1\text{ atm}}{101.325\text{ kPa}}\right)}{(0.75)\left(0.082\frac{\text{L-atm}}{\text{mol-K}}\right)(298\text{ K})}$$

$$C = 0.0999\frac{\text{mol}}{\text{L}}$$

Since there is one 1L in each compartment, there's 0.0999 mol in each compartment. Hence:

$$0.099 \text{ mol} \times \frac{180 \text{ g Mannose}}{1 \text{ mol}} = 17.982 \text{ g of Mannose}$$

B. If the effective resistance to flow across the membrane is 83.5 kPa-s/mL, what is the flow rate of water across the membrane during steady state (in mL/s)? **(10)**

Solution: Remember that flow rate as a function of resistance is given by $Q = \frac{\Delta P}{R}$. You were provided the osmotic pressure gradient (i.e. pressure difference) in part A. The resistance is also provided. Hence, solving for flow:

$$Q = \frac{\Delta P}{R} = \frac{167 \text{ kPa}}{83.5 \ \frac{\text{kPa-s}}{\text{mL}}}$$

$$Q = 2 \ \frac{\text{mL}}{\text{s}}$$

QUESTION 2

Consider a newly discovered microorganism and the transport of hypothetical ions A^+, B^{2+}, C^-, D^-. The table below lists the intracellular and extracellular concentration of each ion as well as each ion's relative conductance. The temperature is 37°C. **(20 total)**

A. Calculate the values of the equilibrium potential for A^+ and B^{2+} (in mV) **(10)**:

Solution: To calculate equilibrium potentials for individual ions, we use the Nernst equation, which is given by:

$$E = -2.3\frac{RT}{zF}\log_{10}\frac{C_i}{C_o}$$

At 37°C, this equation simplifies as follows:

$$E = \frac{-60\text{ mV}}{z}\log_{10}\left(\frac{C_i}{C_o}\right)$$

For ion A^+:

$$E_A = \frac{-60\text{ mV}}{+1}\log_{10}\left(\frac{50\text{ mEq}}{5\text{ mEq}}\right) = -60\text{ mV}$$

Similarly, for ion B^{2+}:

$$E_B = \frac{-60\text{ mV}}{+2}\log_{10}\left(\frac{0.01\text{ mEq}}{1\text{ mEq}}\right) = 60\text{ mV}$$

B. If the overall membrane potential at steady state is -40 mV, estimate the value of the equilibrium potential of C^-. **(10)**

Solution: To the find the equilibrium potential of C^-, we need to use the chord conductance equation:

$$E_m = \frac{g_A}{g_T}E_A + \frac{g_B}{g_T}E_B + \frac{g_C}{g_T}E_C + \frac{g_D}{g_T}E_D$$

From the table, we know that the equilibrium potential of D^- is 0 mV since the intracellular and extracellular concentrations are identical. Hence:

$$E_m = \frac{g_A}{g_T}E_A + \frac{g_B}{g_T}E_B + \frac{g_C}{g_T}E_C$$

$$-40\ \text{mV} = \frac{1}{5}(-60\ \text{mV}) + \frac{1.5}{5}(60\ \text{mV}) + \frac{2.5}{5}E_C$$

$$-40\ \text{mV} = -12\ \text{mV} + 18\ \text{mV} + 0.5E_C$$

$$E_C = -92\ \text{mV}$$

QUESTION 3

Consider a spherical lipid vesicle of membrane thickness 10^{-9} m and diameter of 10^{-6} m that encapsulates a 100mM solution of glucose (MD ~ 10^{-9} m, MW of 180 g/mol) and is placed in a 25°C bath containing 2 mM glucose. Assume that the interior and exterior of the lipid vesicle are osmotically matched, such that the osmotic pressure across the membrane is zero. **(20 total)**

A. If the partition coefficient, k, is 0.0015 and the viscosity, η, is 0.001 N-s/m^2, calculate the steady state flow rate of glucose across the membrane (in mol/s). **(10)**

Solution: The steady state diffusion rate is given by the following equation:

$$J = PA\left(C_A - C_B\right)$$

To solve for *J*, we must first solve for the permeability, *P*:

$$P = \frac{kD}{\Delta x}$$

where $D = \dfrac{KT}{6\pi r\eta} = \dfrac{\left(1.38\times10^{-23}\,\dfrac{\text{J}}{\text{K}}\right)(298\text{K})}{6\pi\left(5\times10^{-10}\ \text{m}\right)\left(0.001\dfrac{\text{N-s}}{\text{m}^2}\right)} = \dfrac{4.363\times10^{-21}\,\text{J}}{9.424\times10^{-12}\,\dfrac{\text{N-s}}{\text{m}}} = 4.363\times10^{-10}\,\dfrac{\text{m}^2}{\text{s}}.$

Hence:

$$P = \frac{kD}{\Delta x} = \frac{(0.0015)\left(4.363\times10^{-10}\,\dfrac{\text{m}^2}{\text{s}}\right)}{10^{-9}\,\text{m}} = 0.000654\,\frac{\text{m}}{\text{s}}$$

Now that we know *P*, we can find *J*:

$$J = PA\left(C_A - C_B\right)$$

$$J = 0.000654\,\frac{m}{s}\left(4\pi\cdot\left(5\times10^{-7}\ \text{m}\right)^2\right)[100\ \text{mM}\ -\ 2\ \text{mM}]\left(\frac{1\ \text{mol}}{1000\ \text{mmol}}\right)\left(\frac{1000\ \text{L}}{1\ \text{m}^3}\right)$$

$$J = 2.013\times10^{-13}\,\frac{\text{mol}}{\text{s}}$$

B. Suppose the glucose was replaced inside and outside by a second molecule V, whose diffusion coefficient is twice that of glucose's but whose partition coefficient is $^1/_3$ that of glucose's. What would be the steady state flow across the membrane? **(10)**

Solution: Reconsider the equation for net rate of diffusion where J_1 is the original rate of diffusion of glucose and J_2 is the second molecule V :

$$J = PA\left(C_A - C_B\right)$$

$$J_1 = \frac{kD}{\Delta x}\left(C_A - C_B\right)$$

$$J_2 = \frac{\left(\frac{k}{3}\right)(2D)}{\Delta x}\left(C_A - C_B\right)$$

$$J_2 = \frac{2}{3}\left[\frac{kD}{\Delta x}\left(C_A - C_B\right)\right]$$

$$J_2 = \frac{2}{3}J_1$$

Hence, $J_2 = \frac{2}{3}J_1 = \frac{2}{3}\left(2.013\times10^{-13}\ \frac{\text{mol}}{\text{s}}\right) = 1.34\times10^{-13}\ \frac{\text{mol}}{\text{s}}$

QUESTION 4

A 35-year old woman comes to your clinic for a routine checkup. Her vital signs are as follows: T=37 ºC, HR=72 beats/min, BP 125/65 mmHg. **(30 total)**

A. What are this woman's pulse pressure and mean arterial pressure (in mmHg)? **(10)**

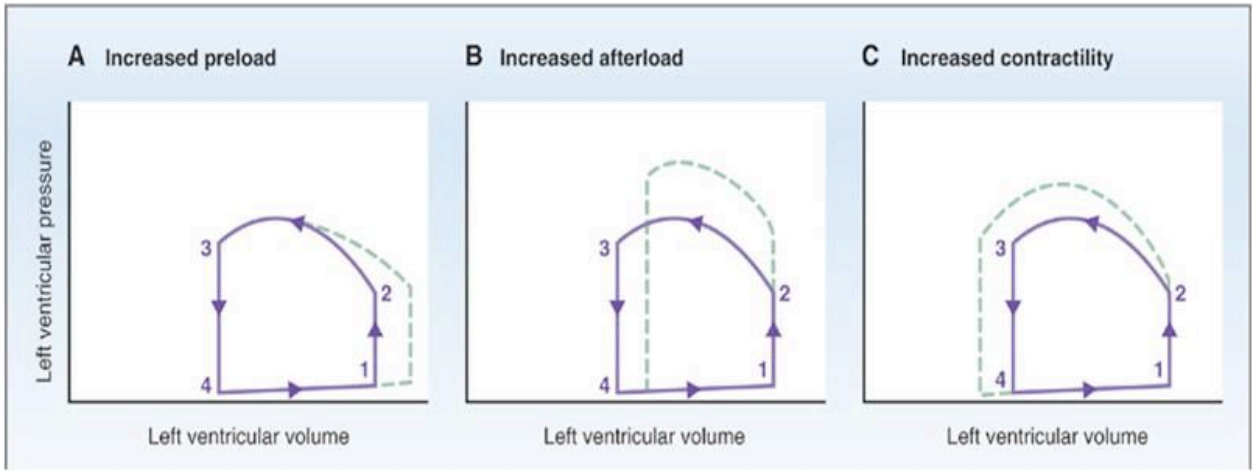
Solution: From the given blood pressure, we know that SP = 125 mmHg and DP = 65 mmHg. PP = SP – DP, so the pulse pressure = 125-65 = 60 mm Hg. MAP = DP + (1/3) PP, so MAP = 85 mm Hg.

B. If this woman has a left ventricular end-diastolic volume of 103 mL and an ejection fraction of 60%, what is her cardiac output? **(10)**

Solution: We are given that LVEDV = 103 mL and EF = 60%. $EF = \frac{SV}{LVEDV}$, $so\ 0.6 = \frac{SV}{103}$, so SV = 61.8 mL. CO = SV * HR = $61.8\frac{mL}{beat} * 72\frac{beats}{\text{minute}}$ = 4449.6 mL/min, or ~ 4.45 L/min.

C. Sketch a representative pressure-volume loop for this patient's left ventricle (don't worry about the actual numbers). Superimposed on this PV loop, draw a new PV loop as a dashed line showing how the PV loop would eventually change if the patient took a positive inotropic agent (e.g. digitalis). Be especially clear what happens to the height, width, and position of the PV loop, and if one or more of these quantities change, explain why in a few words. **(10)**

Solution:



The figure above is Figure 4-24 from the textbook. The main change resulting from a positive inotropic agent is increased contractility (defined on page 139 of the textbook), which shifts the PV loop as shown in panel C. The position of the LVEDV remains the same, but the height and width of the loop increase. The height increases because during the ejection during systole, the ventricle is able to reach a higher pressure

(because increased contractility means that the myocardial muscle cells are able to develop more force). The ventricle is also able to squeeze harder, resulting in a lower End Systolic Volume, which means the loop is wider.
Overall, the new PV loop shows an increase in stroke volume (and CO), which translates to an increase in EF.

QUESTION 5

Consider a small artery of diameter 1.5 cm that simultaneously branches into three arterioles with diameters of 0.5 cm, 0.75 cm, and 1 cm. **(45 total)**

A. If the blood flow through the small artery is 50 ml/min, at what velocity does blood flow through that artery? **(10)**

Solution: $\boldsymbol{v = \frac{Q}{A}}$, where Q = the flowrate and A = the cross-sectional area.

$A = \left(\frac{1.5}{2}\right)^2 \pi = 1.757\ cm^2, and\ v = 50\frac{mL}{minute} * \frac{1}{1.757}\left(\frac{1}{cm^2}\right) * \left(\frac{1cm^3}{mL}\right) = 28.3\frac{cm}{minute}.$

B. What is the flow rate (ml/min) of blood through the largest of the three arterioles? **(15)**

Solution: To find out the flow rate in the arteriole, we need to figure out how the blood flow is split up when the artery branches. We can do this by comparing the resistances of the different arterioles.

From Poiseuille's Law, we know that resistance is inversely proportional to the radius to the 4th power: $R = \frac{8\eta l}{\pi r^4}, R \propto \frac{1}{r^4}$. Although we don't know the actual resistance of any of the vessels, we can still find the ratios between them. Let R_n = resistance of a vessel with *n* cm diameter.

$$\frac{R_{0.75}}{R_1} = \frac{\frac{1}{0.375^4}}{\frac{1}{0.5^4}} = 3.16$$

$$\frac{R_{0.5}}{R_1} = \frac{\frac{1}{0.25^4}}{\frac{1}{0.5^4}} = 16$$

So $R_{0.75} = 3.16*R_1$, and $R_{0.5} = 16*R_1$.

Due to conservation of mass, the flows in the branched vessels must add up to equal the original flow.

$$Q_{tot} = 50\frac{mL}{minute} = Q_{0.75} + Q_1 + Q_{0.5}$$

Substitute using $Q = \frac{\Delta P}{R}$, using resistances in terms of R_1. We assume that the pressure drop stays the same over all the branched arterioles.

$$Q_{tot} = (\Delta P) * \left(\frac{1}{R_{0.5}} + \frac{1}{R_{0.75}} + \frac{1}{R_1}\right) = \Delta P * \left(\frac{1}{R_1} + \frac{1}{3.16 * R_1} + \frac{1}{16 * R_1}\right) = \Delta P * \frac{1.37896}{R_1}$$

$$50\frac{mL}{minute} = \frac{\Delta P}{\frac{R_1}{1.37896}}, so\ \Delta P = 36.26 * R_1, \text{and } Q_1 = \frac{\Delta P}{R_1} = \frac{(36.26\ R_1)}{R_1} = 36.26\frac{mL}{minute}.$$

C. Suppose one of the arterioles eventually branches into a set of capillaries. Consider one of these capillaries: If the oncotic pressures in the capillary and interstitial space are 28 mmHg and 5 mmHg respectively, the hydrostatic pressures in the capillary and interstitial space are 3 mmHg and 32 mmHg respectively, and the flow rate of fluid across the capillary wall is 2.5 ml/min, what is the hydraulic conductance (in ml/min-mmHg)? **(10)**

Solution: Using Starling's Equation, $J_v = K_f[(P_c - P_i) - (\pi_c - \pi_i)]$, we can plug in the flowrate and the various pressures to get:

$$-2.5\,\frac{mL}{minute} = K_f * (-52\ mmHg), so\ K_f = 0.048\,\frac{mL}{min-mmHg}.$$

Note that K_f is positive because it doesn't make sense to have a negative conductance. The question doesn't give a direction for the flow term, but it must be negative because a) the pressure term is negative and K_f can't be, and more importantly, b) if you look at the pressures, the hydrostatic pressure is higher in the interstitial space, which would tend to push flow from the interstitial space into the capillary (negative flowrate). The capillary oncotic pressure is also higher, which tends to pull flow in, so just by looking at the pressures, you could guess that the flowrate will probably be negative.

D. Suppose you do an experiment in which you tie off the small artery at both ends, inject defined volumes of saline solution, and measure the resulting pressure difference across the arterial wall. You find that when the total volume of saline solution in the artery is 50 ml, the transmural pressure is 10 mmHg, and when you inject an additional 25 ml of solution, the pressure increases to 260 mmHg. What is the compliance of the artery (in ml/mmHg)? **(10)**

Solution: Compliance $= \frac{\Delta V}{\Delta P} = \frac{25}{260-10} = 0.1\,\frac{mL}{mmHg}$.

QUESTION 6

Answer the following questions in 1-2 sentences, unless otherwise specified. **(30 total)**

A. Rank conduction speed through these three structures in order from fastest to slowest (no explanation needed): atrial intranodal tracts, atriventricular node, Purkinje fibers. **(5)**

Solution: Purkinjie fibers, atrial intranodal tracts, AV node

B. During phase 4 (rest) of a ventricular action potential, the inward sodium and calcium currents are large enough to balance the outward potassium current, even though the conductance of the membrane to sodium and calcium are much, much smaller than the potassium conductance. How is this possible? **(5)**

Solution: The driving force for the sodium and calcium currents in much larger than the driving force for the potassium current because the membrane potential is close to E_{K+} and very far from E_{Na+} and E_{Ca2+}.

C. Which segment of the EKG would you expect to be most affected by a lesion that delays conduction through the atrioventricular node? **(5)**

Solution: The PR segment corresponds to AV conduction. It's part of the PR interval, which is also an acceptable answer. The length of the PR interval correlates with speed of conduction through the AV node.

D. Carotid sinus massage is often a simple yet effective way of ending episodes of supraventricular tachycardia. Why? **(5)**

Solution: Carotid sinus massage changes baroreceptor signaling (in this case, by simulating more stretch, aka higher BP, and increased firing). The signal would go through the brainstem, leading to increased parasympathetic input to the heart. This would tend to decrease heart rate, which could resolve the tachycardia.

E. Left ventricular hypertrophy is often observed as a compensatory mechanism to chronic systemic hypertension. Use Laplace's law to explain this finding. **(5)**

Solution: Laplace's Law states that P = 2HT/R, so wall pressure is proportional to wall thickness (H). If blood pressure is always high, as in chronic hypertension, the ventricle wall pressure is also higher because more pressure is needed to push blood into the aorta. To support the higher ventricle heart pressure, the heart undergoes remodeling to increase the wall thickness – the cells in the wall become larger (hypertrophy).

F. When a neuron in the body fires an action potential, why does it only travel in one direction? Your answer should specify a molecular mechanism **(5)**

Solution: Action potentials only travel in one direction because the neurons undergo a refractory period immediately after firing. There is an absolute refractory period, which is due to closure of inactivation gates on Na+ channels—these gates take a while to

reopen, so when they are closed depolarization is impossible. Afterwards, there is also a relative refractory period during which the cell is hyperpolarized (after a large K+ current). During this period, action potentials are possible, but they are more unlikely because it would take a large amount of depolarization to reach the threshold for the potential.