Math 1A Second Midterm

Thu 1 Nov 2012, 11:10–12:30 PM

Your Name: ______________________

Circle your section number:

Section	Time	GSI
201	MWF 8–9	Raymond Christopher
202	MWF 8–9	Jakub Kominiarczuk
203	MWF 9–10	Jakub Kominiarczuk
204	MWF 9–10	Xin Jin
205	MWF 10–11	Xin Jin
206	MWF 11–12	Khoa Nguyen
207	MWF 12–1	Khoa Nguyen

Section	Time	GSI
208	MWF 1–2	Thunwa Theerakarn
209	MWF 2–3	Thunwa Theerakarn
210	MWF 4–5	Andrew Dudzik
211	MWF 4–5	Raymond Christopher
212	MWF 5–6	Andrew Dudzik
213	MF 10–12	Yuhao Huang

Instructions

(1). Check that you have all 8 pages of this exam booklet.

(2). Be sure to show all your steps. *In particular, a "yes" or "no" or numerical answer by itself is never sufficient.* When in doubt, over-explain rather than under-explain.

(3). Calculators are not allowed.

(4). Check your work as time allows.

(5). You may not use anything that has not been covered in the course or its prerequisites.

EXAM SCORES		
Problem	Max	Your Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
Total	0	

1. (12 points) Calculate y'.

(a). $y = \ln\left|\tan^{-1} x\right|$

(b). $y = e^{\sin^4(x^2)}$

2. (12 points) Use logarithmic differentiation (fully) to calculate

$$\frac{d}{dx}\left(\frac{(x^2+2)^3(x-6)^{17}x^x}{e^x\sqrt{2-x^2}}\right).$$

3. (18 points) A Halloween monster is shaped like a circular cone. It is undergoing a transformation in which its height is decreasing at the rate of 1 cm/sec, while its volume is decreasing at the rate of 4π cm^3/sec. (It retains its conical shape while this is happening.) If, at a given instant, its volume is 15π cm^3 and its height is 5 cm, determine whether its radius is increasing or decreasing at that instant, and at what rate.
(The volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height.)

4. (10 points) Use a linear approximation (or differentials) to estimate $\sqrt[4]{1.003}$.

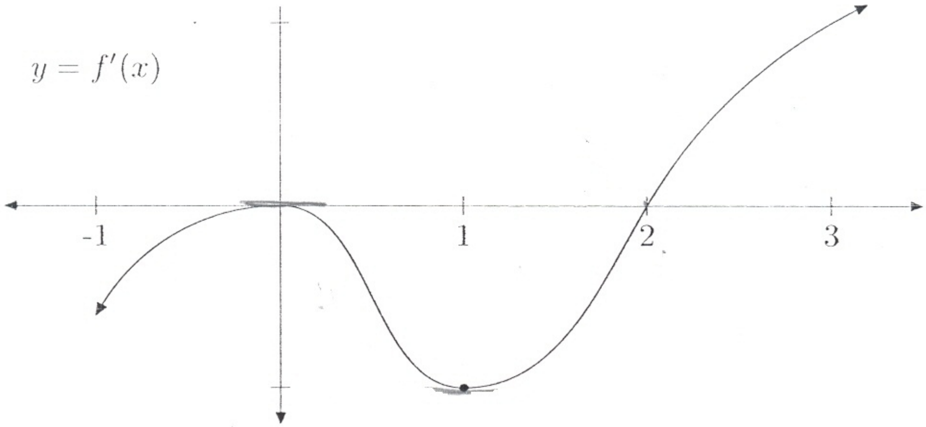
5. (15 points) (a). State the Mean Value Theorem, fully and carefully.

(b). Check that the function $f(x) = \sin^{-1} x$ satisfies the conditions of the theorem for the interval $[-1, 1]$, and find all values of c that satisfy the conclusion of the theorem.
(For the first part, you may recite known properties of the arcsine function without deriving them again.)

6. (15 points) Show that the function $f(x) = x + \cos x$ with domain $[0, 3\pi]$ is a one-to-one function.

[**Hint:** Think about what its graph looks like.]

7. (18 points) The graph of the *derivative* f' of a differentiable function $f : \mathbb{R} \to \mathbb{R}$ is shown.



(a). On what intervals is f increasing or decreasing? (Give the largest intervals possible.) Where does f have local maxima and minima?

(b). On what intervals is f concave up or concave down? Where does f have points of inflection?

(c). Does f have an absolute maximum? If so, where does it occur? Does f have an absolute minumum? If so, where does it occur?