QUIZ 3 SOLUTION - Chiffre

Problem 1 (20 pts)

A compressor has two inlets and one outlet:

Input 1 takes in saturated liquid water at T_1=70 °C and $\dot{m}_1$ =5 kg/s

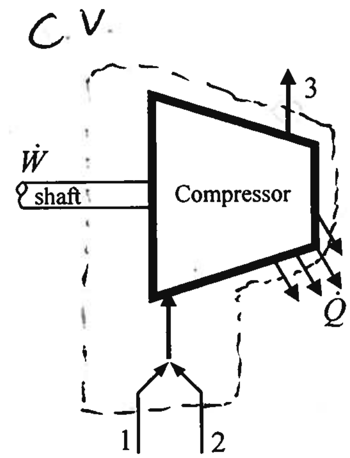
Input 2 takes in saturated water vapor at T_2=70 °C and $\dot{m}_2$ =20 kg/s.

The outlet is at p_3= 800 kPa.

The compressor requires $\dot{W}$ = 60 MW (60 $\times 10^6$ W) of work input.

The compressor is uninsulated, and there is a heat loss of 10 MW.

Changes in kinetic and potential energy are negligible.



(a) Obtain a symbolic equation for the enthalpy at the exit. Do not plug in any numbers.

(b) What is the numerical value of temperature of the exiting stream, T_3?

(a)

MASS: $\cancel{\frac{d}{dt} M_{cv}} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2$

S. S.

ENERGY: $\cancel{\frac{d}{dt} E_{cv}} = \underset{NF}{\dot{W}_{in}} - \dot{Q}_{out} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$

S. S.

$$h_3 = \frac{\underset{NF}{\dot{W}_{in}} - \dot{Q}_{out} + \dot{m}_1 h_1 + \dot{m}_2 h_2}{\dot{m}_1 + \dot{m}_2}$$

(b) Need h_1 : $h_f(70°C) = 293.1$ kJ/kg $\rightarrow \dot{m}_1 h_1 = 1465.5$ kW

h_2 : $h_g(70°C) = 2626.1$ " $\rightarrow \dot{m}_2 h_2 = 52,522$ kW

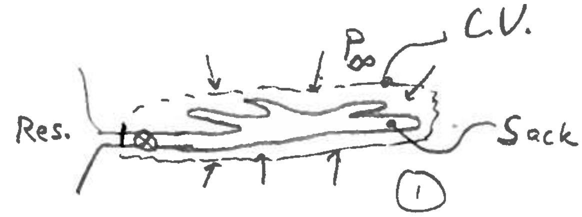
$\Rightarrow h_3 = 4159.5 \frac{kJ}{kg}$

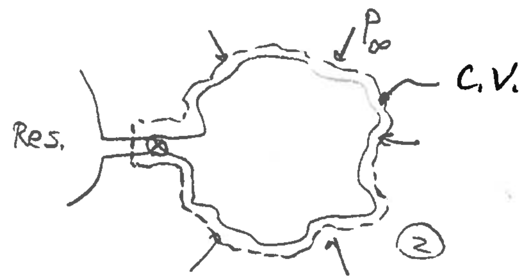
and $P_3 = 800$ kPa $= 0.8$ MPa.

S.H.V. Table $\rightarrow T_3 \approx 800°C$ $\left(h = 4157 \frac{kJ}{kg}\right)$

Problem 2 (20 pts)

A thin, flexible sack is connected by a valve to a reservoir of saturated water vapor at 120 °C. Initially the sack is completely empty. The valve is opened slightly and steam begins to fill the sack. After some time the sack volume has expanded to 0.50 m^3. Weighing the sack reveals that its mass has increased by 0.23 kg. The pressure outside the sack is p_∞=100 kPa throughout this process. There was also a heat transfer Q_{12} (value unknown) between sack and surroundings during this process.





(a) What is the final temperature of the water vapor inside the sack?

(b) What was the heat transfer $Q_{12,}$ in kJ? Clearly specify the magnitude and direction.

MASS: $M_{cv_2} - \overset{\text{empty}}{\cancel{M_{cv_1}}} = m_{i_{12}} - \overset{\text{no exit}}{\cancel{m_{e_{12}}}} \Rightarrow M_{cv_2} = m_{in_{12}}$

ENERGY $E_{cv_2} - \underset{\text{empty}}{\cancel{E_{cv_1}}} = Q_{12,in} - W_{12,\text{out NF}} + \underbrace{m_i h_i} - \cancel{m_e} h_e$

pdV work.

(or $\int_1^2 \dot{m}_i h_i \, dt$, but $h_i = h_{Res}$ = const.)

$$M_{cv_2} u_2 = Q_{12,in} - P_\infty (V_2 - \underset{0}{\cancel{V_1}}) + m_i h_i$$

$$Q_{12,in} = M_{cv_2}(u_2 - h_i) + P_\infty M_{cv_2} v_2$$

$$\boxed{Q_{12,in} = M_{cv_2}[h_2 - h_i]}$$

$h_i = h_g(120°C) = 2706.0 \frac{kJ}{kg}$

Need h_2. State 2: $p = P_\infty = 100$ kPa

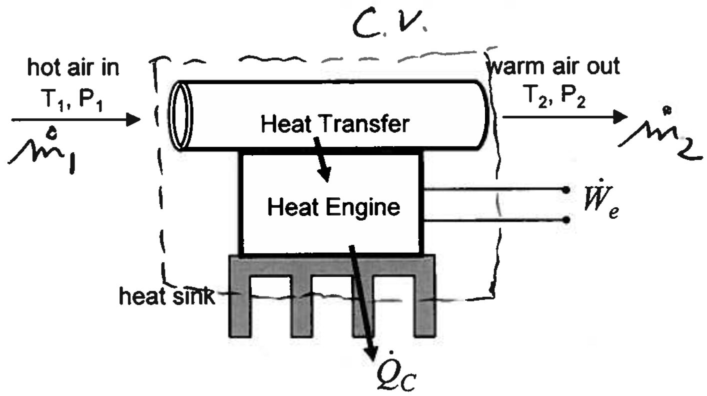
$$v = \frac{V_2}{m_2} = \frac{0.5 m^3}{0.23 kg} = 2.17 \frac{m^3}{kg}.$$

$\Rightarrow$ SHV at $\boxed{200°C = T_2}$, $h_2 = 2875.5 \frac{kJ}{kg}$

$$\boxed{Q_{12,in} = +39.0 \text{ kJ}}$$

Problem 3 (20 pts)

(Background: devices like this are being considered to generate electricity from hot car exhaust.)



Hot air enters a pipe at T_1, P_1, and exits at T_2, P_2. A heat engine is mounted to the side of the pipe, such that there is a steady heat transfer from the pipe to the heat engine. The heat engine generates electrical power $\dot{W}_e$. The heat engine also rejects heat to the surroundings at a rate $\dot{Q}_C$

Given:

T_1, P_1, T_2, P_2, $\dot{W}_e$, $\dot{Q}_C$, Air as an ideal gas, with constant C_p, C_v, and R

Derive an expression for the mass flow rate of air, $\dot{m}$. You may ignore kinetic energy effects.

MASS: $\cancel{\frac{d}{dt} M_{cv}} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2$

S.S

ENERGY: $\cancel{\frac{d}{dt} E_{cv}} = -\dot{Q}_c - \dot{W}_e + \dot{m}_1 h_1 - \dot{m}_2 h_2$

S.S.

$$\dot{m}(h_1 - h_2) = \dot{W}_e + \dot{Q}_c$$

Ideal gas, const. C_p: $\Delta h = C_p \Delta T$

$$\boxed{\dot{m} = \frac{\dot{W}_e + \dot{Q}_c}{C_p (T_1 - T_2)}}$$

(...more space for Problem 3)

$\boxed{E.C.}$ Now add KE terms to $\dot{m}$

$$0 = -\dot{Q}_c - \dot{W}_e + \dot{m}\left[\left(h_1 + \tfrac{1}{2}V_1^2\right) - \left(h_2 + \tfrac{1}{2}V_2^2\right)\right]$$

$$\cancel{\dot{m} = \frac{\dot{W}_e + \dot{Q}_c}{c_p(T_1 - T_2) + \frac{1}{2}(V_1^2 - V_2^2)}}$$

* Which is faster, V_1 or V_2?

Mass: $\dot{m} = \dfrac{A_c V}{v}$ IGL: $Pv = RT$

$$v = \frac{RT}{P} \rightarrow V = \frac{\dot{m}}{A_c}\frac{RT}{P}$$

(Extra Credit: 5 pts) If we did account for changes in kinetic energy, qualitatively how would the work output change? Other than $\dot{W}_e$ and KE, assume all quantities given above remain constant. *You must justify your answer.*

(i) Accounting for KE effects would cause $\dot{W}_e$ to **increase**.

(ii) Accounting for KE effects would cause $\dot{W}_e$ to **stay the same**.

(iii) Accounting for KE effects would cause $\dot{W}_e$ to **decrease**

$$\dot{W}_e = -\dot{Q}_c + \dot{m}\,c_p(T_1 - T_2) + \frac{\dot{m}}{2}\left[\left(\frac{\dot{m}}{A_c}\frac{RT_1}{P_1}\right)^2 - \left(\frac{\dot{m}R}{A_c}\frac{T_2}{P_2}\right)^2\right]$$

If $\dfrac{T_1}{P_1} > \dfrac{T_2}{P_2}$: $\dot{W}_e$ incr.

If $\dfrac{T_1}{P_1} < \dfrac{T_2}{P_2}$: $\dot{W}_e$ decr.

Need more info to decide!

Realistically, $\dfrac{\Delta P}{P_{avg}}$ typically a few %, whereas $\dfrac{\Delta T}{T_{avg}} \gtrsim 10\%$

so ΔT effect most likely to dominate.

Expect $\dot{W}_e$ increase, (most likely)