EE 20N: Structure and Interpretation of Signals and Systems

Midterm 2

12:40- 2:00, April 4, 2013

Notes: There are **eight** *questions on this midterm. Answer each question part in the space below it, using the back of the sheet to continue your answer if necessary. You can use any facts in the lecture, lab and discussion notes without deriving them again.* **None of the questions requires a very long answer, so avoid writing too much! Unclear or long-winded solutions may be penalized.** *The approximate credit for each question part is shown in the margin (total 45 points).*

Your Name: **Your Lab Section:**

Name of Student on Your Left:
Name of Student on Your Right:

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	

Problem 6	
Problem 7	
Problem 8	
Total	

1. (4 points) Write down the 3-point DFT and IDFT in matrix form. The entries of the matrices involved should be written as complex numbers in rectangular form (i.e. $a + bi$).

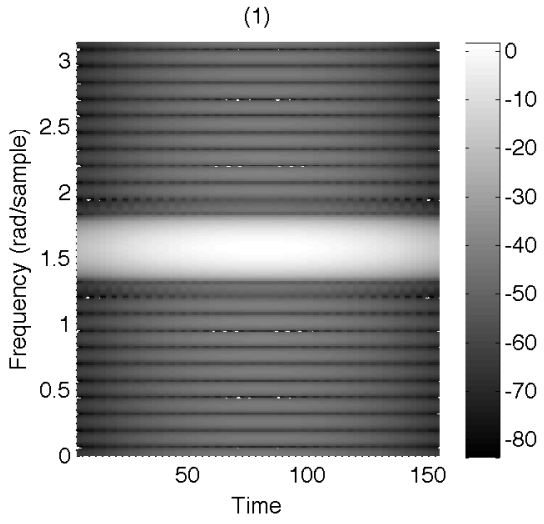
2. (4 points) Match the following signals to their respective spectrograms. Assume all signals are of duration 1000 samples.

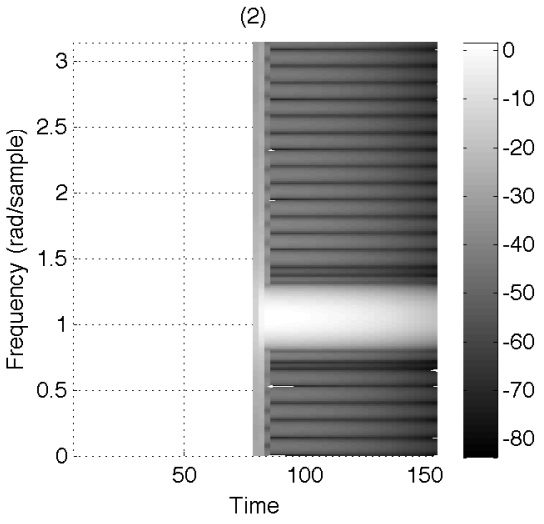
   a) $x(n) = 1 + \cos(\frac{\pi}{2}n)$

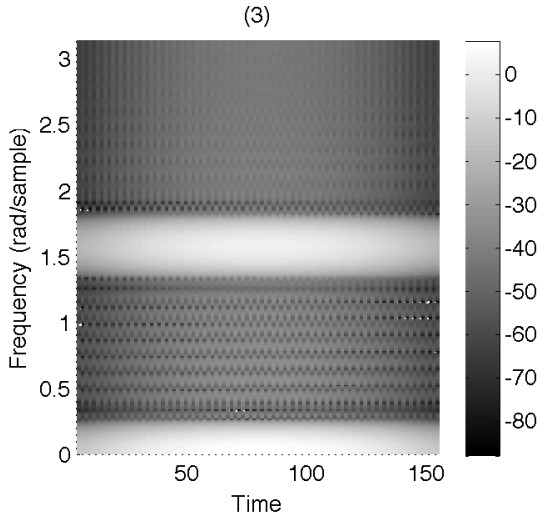
   b) $x(n) = \sin(\frac{\pi}{2}n)$

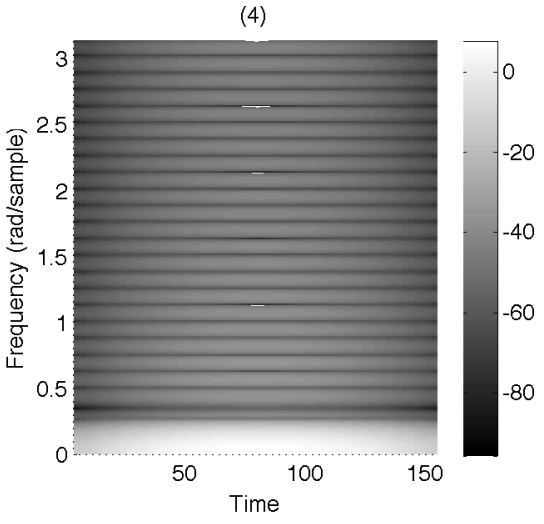
   c) $x(n) = 1$

   d) $x(n) = \begin{cases} 0 & : n < 500 \\ \cos(\frac{\pi}{3}n) & : n \geq 500 \end{cases}$



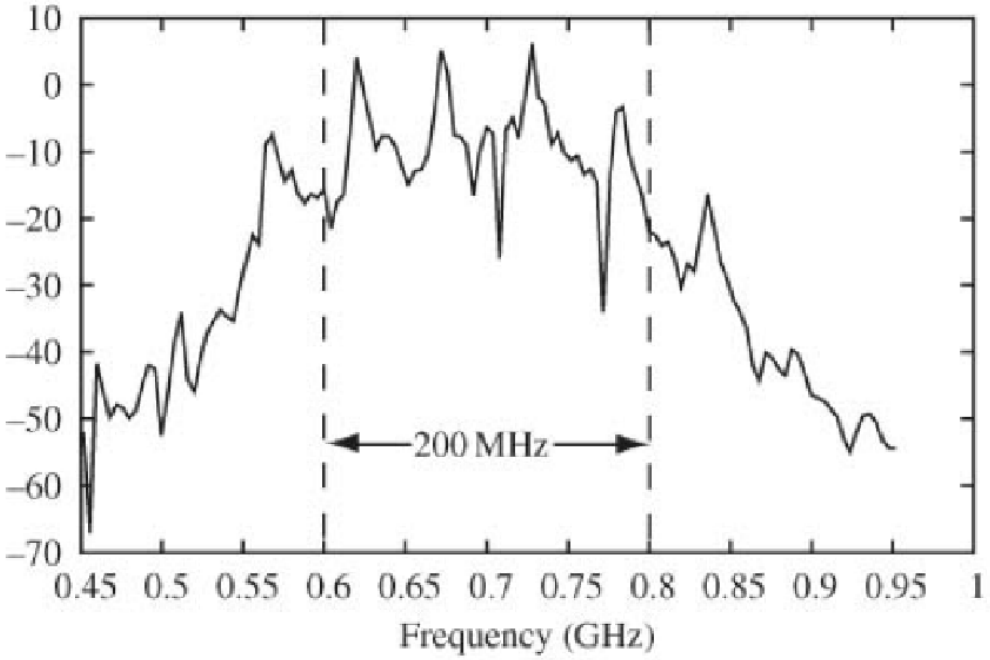






3. (5 points) Suppose the response of a discrete-time LTI system to a step input is $g(n)$ (the so-called *step response*). Here, a step input is the signal $u(n) = 0$ for $n < 0$ and $u(n) = 1$ for $n \geq 0$. Does the step response $g(n)$ fully specify what the LTI system is? If so, compute the impulse response $h(n)$ of the system in terms of $g(n)$. If not, give an example of two LTI systems having the same step response.

4. (6 points) Consider the frequency response of a wireless channel shown below. The magnitude is plotted on the dB scale. We restrict ourselves to within the 200 MHz band.



a) (1 point) At what frequency is the channel strongest?

b) (1 point) At what frequency is the channel weakest?

c) (2 points) Roughly by what factor is the strongest channel stronger than the weakest channel? (an order-of-estimate is fine)

d) (2 points) Consider an OFDM system using 8 sub-carriers (i.e. using an 8-point DFT). What is the frequency (in Hz) of the sub-carrier having the strongest channel?

5. (7 points) You want to build a system to generate music with certain frequency components . You have at your disposal:

   - p-point DFT blocks, for $p = 128, 512$ and 1024.
   - p-point IDFT blocks, for $p = 128, 512$ and 1024.
   - D/A converters at sampling frequencies $f_s = 2.5, 10$ and 20 k-samples/s.
   - A/D converters at sampling frequencies $f_s = 2.5, 10$ and 20 k-samples/s.

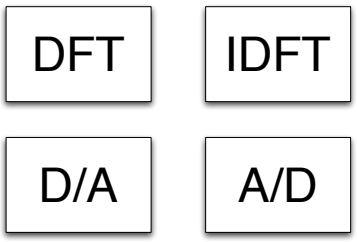
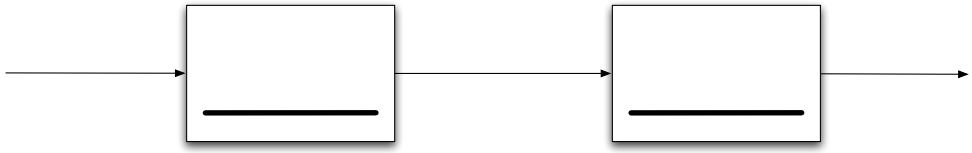


Figure 1: Available blocks

a) (2 points) Build your system by filling in each of the blank blocks in the system diagram below by one of the available blocks above.



b) (5 points) Choose values for the block size p, the sampling frequency f_s and the input to your system to generate at the output music with frequency components at approximately 880 Hz and 1760 Hz for a duration of approximately 50 ms.

6. (6 points) Consider the DT system shown in the figure below.

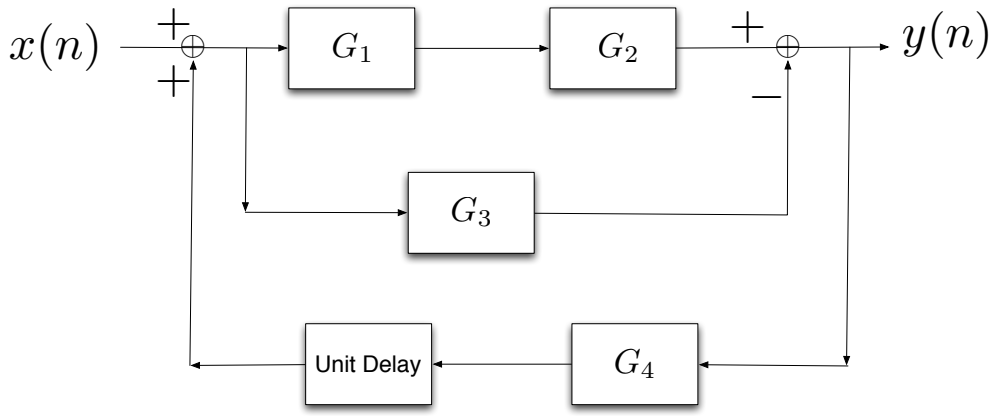


Figure 2: Composition of Systems

a) (4 points) Compute the frequency response H of the overall system in terms of the frequency responses of the subsystems.

b) (2 points) If all the subsystems are causal, must the overall system be causal? (Respond in "yes", "no" or "don't know", no explanation required. A correct answer gets 2 points, incorrect answer gets 0 point, and "don't know" gets 1 point.)

7. (8 points) Consider a CT LTI system with the following input output relationship:

$$y(t) = \int_0^\infty e^{-s/T} x(t-s) ds.$$

a) (4 points) Compute the frequency response of this system. Plot its magnitude as a function of frequency.

b) (1 point) Interpreting this system as a filter, what kind of filter is this?

c) (3 points) What should be the unit of the parameter T? Explain qualitatively what happens to the frequency response as T is varied. Give an interpretation of the parameter T.

8. (5 points) Consider a causal LTI system. The output of the system given a periodic input $x(n)$ with period p is an output $y(n)$. Define:

$$\begin{aligned} \tilde{x}(n) &= \begin{cases} x(n) & n = 0, 1, \ldots, p-1 \\ 0 & \text{else} \end{cases} \\ \tilde{y}(n) &= \begin{cases} y(n) & n = 0, 1, \ldots, p-1 \\ 0 & \text{else} \end{cases} \end{aligned}$$

Is $\tilde{y}(n)$ the output of the (same) LTI system when the input is $\tilde{x}(n)$? If "yes", give a proof. If "no", give a counter-example.