MATH 1B: Spring 2013 — Midterm 1

Name: _____

Student ID: _____

GSI: _____

The exam is closed book, apart from a sheet of notes 8"x11". Calculators are not allowed. You need to show all the reasoning that goes into solving the problem, step by step – the answer alone is not enough. It is your responsibility to write your answers clearly.

There is a single page of problems. Please write solutions in blue books.

Problem 1 _____

Problem 2 _____

Problem 3 _____

Problem 4 _____

Problem 5 _____

Total _____

1. (5 points) *Consider the sequence*

$$a_n = \frac{1}{2^{n^2}} - \frac{1}{2^{(n+1)^2}}$$

ⓐ *Find the limit of the sequence, if it exists.*

ⓑ *Find the value of* $\sum_{n=1}^{\infty} a_n$, *if the series converges. If the series diverges, state why this is the case.*

2. (5 points) *Find the limit of the sequence given by*

$$a_n = (-1)^n n^2 e^{-n}$$

3. (5 points) *Find the integral*

$$\int \frac{1}{t(\ln(t)+3)(\ln(t))^2} dt.$$

4. (5 points) *Evaluate the integral, assuming* $a > 0$ *is a given constant.*

$$\int_{\frac{2a}{\sqrt{3}}}^{2a} \frac{1}{x^3(x^2-a^2)^{1/2}} dx.$$

5. (5 points) *Use the comparison test to determine whether the integral converges or diverges. If the integral converges, estimate its value by giving a lower and an upper bound to the value of the integral.*

$$\int_0^{\infty} \frac{1}{\sqrt{x}(1+e^x)} dx.$$

(Hint: what kind of an improper integral is this?)