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ME173
Fundamentals of Acoustics

Spring 2011

Midterm Exam

Please write you name on the exam (you'd be surprised at how many times we get exams with no names on them …) and clearly identify your answers.

1) A small source operating in air produces 100 mW of acoustic power at a frequency of 100 Hz.

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   a) Determine the following quantities at a distance of 0.5 m from the source:

      i) The amplitude of the acoustic pressure in Pa.
      ii) The amplitude of the acoustic particle velocity in m/s.
      iii) The sound pressure level in dB re 20 μPa.
      iv) The phase angle between the pressure and the particle velocity.

   b) The threshold of hearing at 100 Hz is about 22 dB re 20 μPa. How far away from the source must one be in order for the sound pressure level to be at this value?

   c) The threshold of hearing at 4 kHz is about 0 dB re 20 μPa. How far away from a source producing 100 mW of power at 4 kHz must one be in order for the SPL to be at 0 dB re 20 μPa?

a) $\Pi = 100$ mW $\quad f = 100$ Hz $\quad r = 0.5$ m

i) $\Pi = I \cdot A = I \cdot 4\pi r^2 = \frac{1}{2}\frac{P_+^2}{\rho_0 c} \cdot 4\pi r^2$

$P_+ = \sqrt{\frac{\rho_0 c \Pi}{2\pi r^2}} \qquad \rho_0 c = 415 \frac{Pa \cdot s}{m} \quad (c = 343 \frac{m}{s})$

$\boxed{P_+ = 5.14 \text{ Pa}}$

$= IL = 10 \log_{10} \frac{I}{I_{ref}}$

$= 105$ dB

ii) $Z = \frac{P(r)}{v(r)} = \rho_0 c \left(\frac{kr}{1+(kr)^2}\right)(kr + i)$

$|v(r)| = \frac{P(r)}{|Z|} = \frac{P}{\rho_0 c \left(\frac{kr}{1+(kr)^2}\right)\sqrt{(kr)^2+1}} = \frac{P}{\rho_0 c \frac{kr}{\sqrt{(kr)^2+1}}}$

$k = 1.8 \text{ m}^{-1}$

$\boxed{v_+ = 0.0185 \text{ m/s}}$

iii) $SPL = 20 \log_{10} \frac{P_{rms}}{20 \mu Pa}$ dB $\qquad P_{rms} = \frac{P_+}{\sqrt{2}}$ $\qquad \boxed{SPL = 105 \text{ dB}}$

iv) $\phi = \measuredangle \frac{P}{v} = \measuredangle Z = \tan^{-1}\left(\frac{1}{kr}\right)$ $\qquad \boxed{\phi = 48°}$

b) $SPL_{cr} = 22$ dB $= 20 \log_{10} \frac{P_{rms}}{P_{ref}} \qquad P_{rms} = \frac{P_+}{\sqrt{2}} = \frac{\sqrt{\frac{\rho_0 c \Pi}{2\pi r^2}}}{\sqrt{2}}$

$\sqrt{\frac{\rho_0 c \Pi}{4\pi r^2}} = P_{ref}\, 10^{\frac{22}{20}} \Rightarrow r = \sqrt{\frac{\rho_0 c \Pi}{4\pi}} \Big/ P_{ref} 10^{\frac{22}{20}}$ $\qquad \boxed{r = 7.22 \text{ km}}$

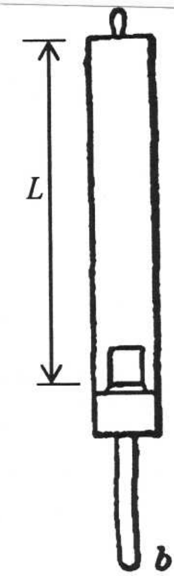
Somewhat unrealistic!

c) $f = 4$ kHz $\quad SPL_{cr} = 0$ dB

$r = \frac{\sqrt{\frac{\rho_0 c \Pi}{4\pi}}}{P_{ref}\, 10^{0/20}}$ $\qquad \boxed{r = 90.9 \text{ km}}$

Again, unrealistic. In reality, there's a lot of attenuation.

2) (a) Determine the length L of a stopped organ pipe (i.e., a closed-end organ pipe) that is tuned to 440 Hz.



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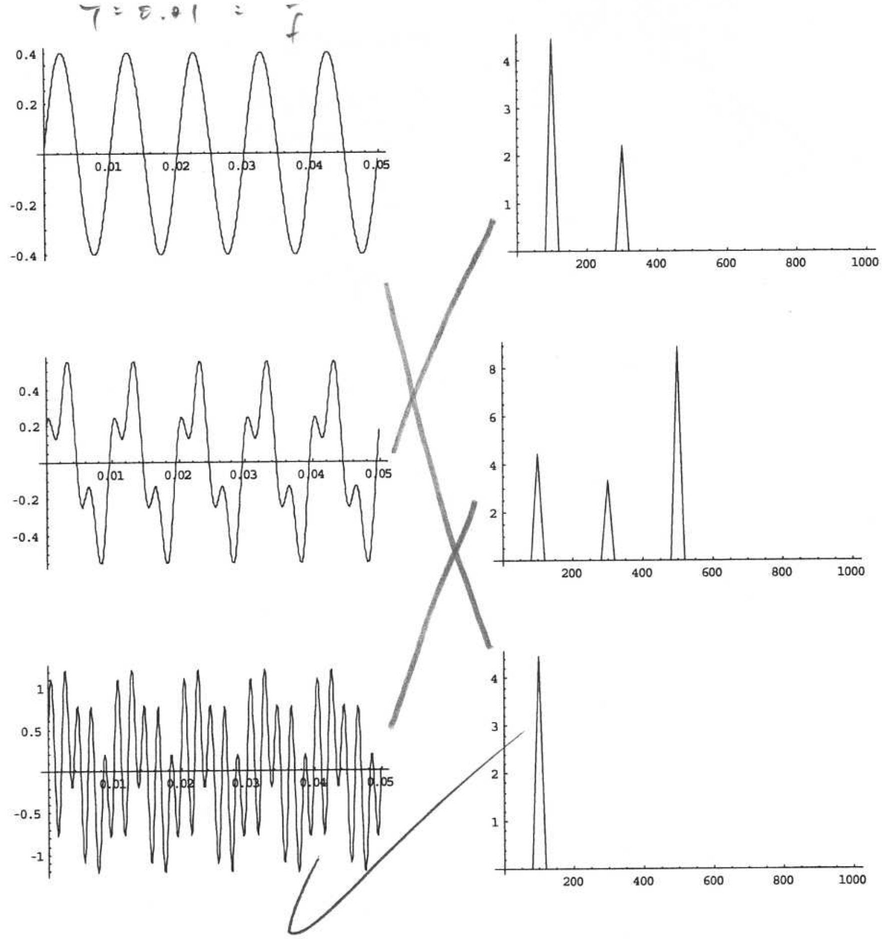
(b) What role, if any, does the diameter of the pipe play in the way that the pipe sounds?

(c) Shown below on the left side of the page are three waveforms (voltage vs. time) with their associated frequency spectra shown on the right side of the page. However, the frequency spectrum for a particular waveform may or may not be next to that waveform. Please draw a line from each waveform to the corresponding spectrum. (Note that the frequencies for these waves is not the same as for the organ pipe that you have just designed.)

$T = 0.01 = \frac{1}{f}$



forced, closed

a) $f_1 = 440 \text{ Hz} = \frac{nc}{4L}$, $n = 1$ $c = 343 \frac{m}{s}$

$L = \frac{c}{4f}$ $\boxed{L = 0.195 \text{ m}}$

b) The pipe diameter should only affect the sound transmission characteristics of the pipe, assuming pressure in the pipe continues to behave like plane waves. A longer diameter should produce a more even sound distribution. ?

~~d~~ se

c) (on other page)

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3) A graduate student wants to use the ultrasonic range finder shown to the right to monitor the amount of water in a large tank. This range finder works by producing a short tone at 42 kHz and then determining the time required for this tone to return to the sensor. The device itself would be placed at the top of the tank pointed downward so that the acoustic signal will travel through the air, reflect off of the water surface and return to the sensor.

The student would like to protect the ultrasonic system from splashes that may occur within the tank, and has proposed putting a thin plastic film over the existing covering (the black plastic cylinder in the photograph). Unfortunately, his initial attempts at covering the sensor have caused the range finder to stop working as it should, perhaps because too much of the signal is being reflected by the thin film.

For this problem, you are to estimate the amount of acoustic energy that is transmitted through the layer for various film thicknesses. "Standard" kitchen plastic wrap (Saran Wrap, or its equivalent) has a thickness of about 12 μm and has a specific acoustic impedance of about 2×10^6 Pa s/m.

a) Determine the amount of energy that is transmitted (as a percent of the amount produced) by a plastic layer with thickness of 12 μm and specific acoustic impedance of 2×10^6 Pa s/m.

b) Determine the film thickness that would allow 95% of the incident energy to be transmitted.

c) Comment on the practical difficulties that might occur in trying to do this, if a sufficiently thin film could be found.

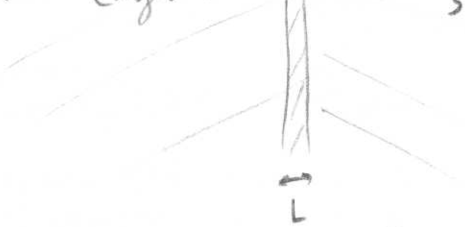
a)  $L = 12\ \mu m$ $Z_1 = 2\times10^6\ \frac{Pa\,s}{m}$ $f = 42\ kHz$

$C_{wrap} = 2500\ \frac{m}{s}$

For transmission through a layer, the transmission intensity coefficient T_i is given in the "Notes on Acoustic Waves in Ideal Fluids":

$$T_i = \frac{4}{2 + \left(\frac{r_3}{r_1} + \frac{r_1}{r_3}\right)\cos^2 k_2 L + \left(\frac{r_2^2}{r_1 r_3} + \frac{r_1 r_3}{r_2^2}\right)\sin^2 k_2 L}$$

for the situation Layer 1 2 3



$r_n = Z_n$

Here, we have $r_1 = r_3 = \rho_{air} c_{air} = 415\ \frac{Pa\,s}{m}$, $r_2 = 2\times10^6\ \frac{Pa\,s}{m}$,

$L = 12\ \mu m$, $k_2 = \frac{\omega}{c_{wrap}} = \frac{2\pi f}{c_{wrap}} = 105.6\ m^{-1}$

$$T_i = \frac{4}{2 + (1+1)\cos^2 k_2 L + \left(\frac{r_2^2}{r_1^2} + \frac{r_1^2}{r_2^2}\right)\sin^2 k_2 L}$$

$$= \frac{1}{1 + \frac{1}{4}\left(\frac{r_2}{r_1} - \frac{r_1}{r_2}\right)^2 \sin^2 k_2 L}$$

$k_2 L = 0.00127$;
$\sin k_2 L = 0.00127$

$T_i = 0.0969$

$E_{trans} = T_i\, E_{\substack{incident \\ produced}}$

$\boxed{E_t = 9.69\%\ E_i}$

(continued)

b) $T_i = 0.95 = \dfrac{1}{1 + \frac{1}{4}\left(\frac{r_2}{r_1} - \frac{r_1}{r_2}\right)^2 \sin^2 k_2 L_1}$

$$\sin^2 k_2 L_1 = -\frac{4}{\left(\frac{r_2}{r_1} - \frac{r_1}{r_2}\right)^2} + \frac{4}{T_i\left(\frac{r_2}{r_1} - \frac{r_1}{r_2}\right)^2}$$

$$\sin k_2 L_1 = \frac{2}{\left(\frac{r_2}{r_1} - \frac{r_1}{r_2}\right)} \sqrt{\frac{1}{T_i} - 1}$$

$$L_1 = \frac{1}{k_2} \sin^{-1}\left(\frac{2}{\left(\frac{r_2}{r_1} - \frac{r_1}{r_2}\right)} \sqrt{\frac{1}{T_i} - 1}\right)$$

$$L_1 = 0.902 \times 10^{-6} \text{ m} \qquad \boxed{L_1 = 0.902\ \mu\text{m}}$$

c) Disregarding the undoubtedly low strength of the resulting material, many difficulties might occur. If the film is stretched, its impedance and thickness characteristics will change, making it a poor quarter-wave plate. Over time, power transmission through the film would likely deform the material despite its quarterwave characteristics. In general, the conditions in reality are too nonideal to ensure proper ~~p~~ acoustic transmission.

(The film would also attenuate the intensity of the returning acoustic wave, if it covers the microphone sensor on the chip.)