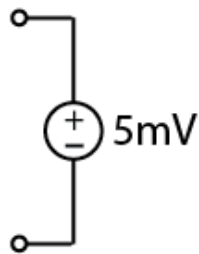
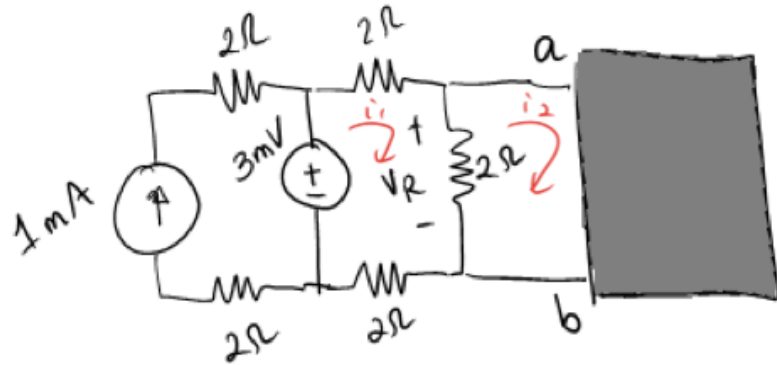


## Midterm 1 Solution, Spring 2013

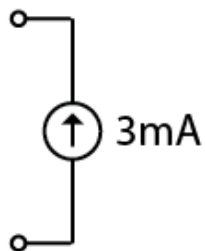
Problem 1:

a)  $R_{TH} = 0$   
 $V_{TH} = \alpha (V_A / (R_1 + R_2))$

b)



or



$$3 - 6i_1 + 2i_2 = 0$$

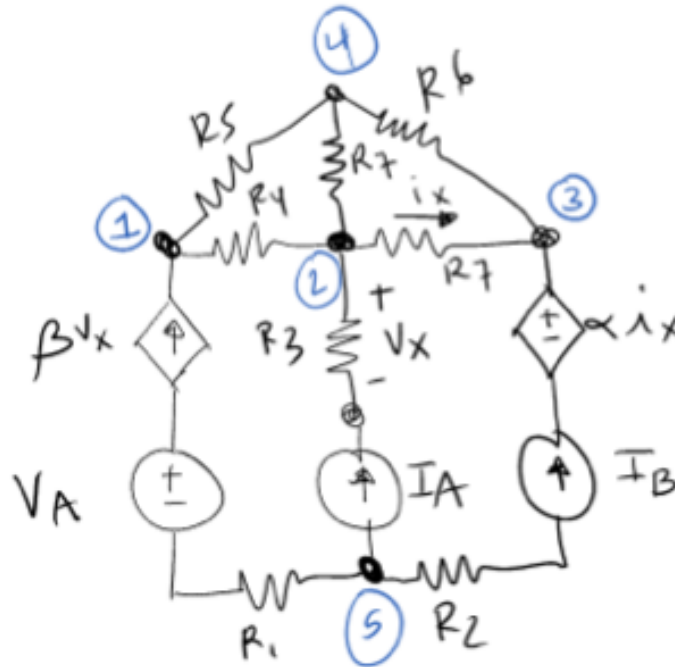
$$2(i_1 - i_2) = 5$$

$$3 - 4i_1 = 5$$

$$i_1 = -0.5 \text{ mA}$$

$$i_2 = i_1 - 2.5 = -3 \text{ mA}$$

Problem 2:

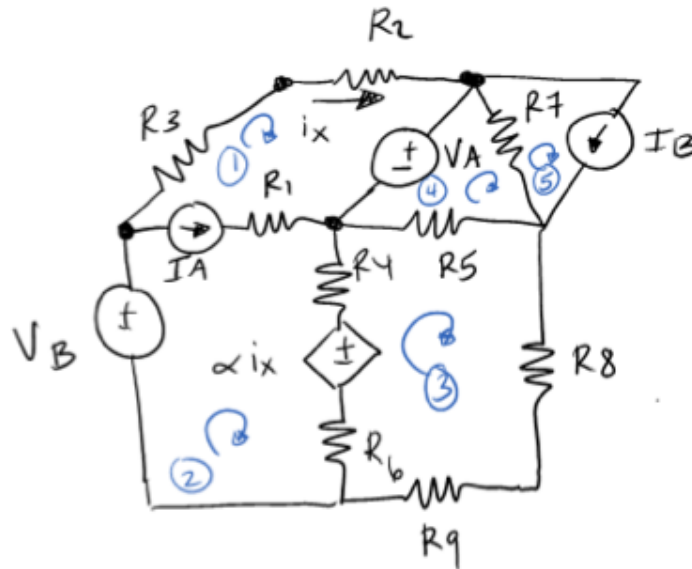


- 1)  $(V_1 - V_2)/R_4 + (V_1 - V_4)/R_5 - \beta V_X = 0$  ,  $V_X = -R_3 I_A$
- 2)  $(V_2 - V_1)/R_4 + (V_2 - V_4)/R_7 + (V_2 - V_3)/R_8 - I_A = 0$
- 3)  $(V_3 - V_4)/R_6 + (V_3 - V_2)/R_8 - I_B = 0$
- 4)  $(V_4 - V_1)/R_5 + (V_4 - V_2)/R_7 + (V_4 - V_3)/R_6 = 0$
- 5)  $\beta V_X + I_A + I_B = 0$      $V_X = -(I_A + I_B) / \beta$

In 1, you could also use 5 to replace  $(-R_3 I_A)$  with  $-(I_A + I_B) / \beta$ , alternatively. Also, you could ground any of the nodes and set it to 0 for that node voltage to simplify the equations further.

- 1)  $(1/R_4 + 1/R_5)V_1 - (1/R_4)V_2 - (1/R_5)V_4 = -\beta R_3 I_A$
- 2)  $-(1/R_4)V_1 + (1/R_4 + 1/R_7 + 1/R_8)V_2 - (1/R_8)V_3 - (1/R_7)V_4 = I_A$
- 3)  $-(1/R_8)V_2 + (1/R_8 + 1/R_6)V_3 - (1/R_6)V_4 = I_B$
- 4)  $-(1/R_5)V_1 - (1/R_7)V_2 - (1/R_6)V_3 + (1/R_5 + 1/R_6 + 1/R_7)V_4 = 0$

Problem 3:



Supermesh 1,2:

$$R_3 I_1 + R_2 I_1 + V_A + R_4 (I_2 - I_3) + \alpha i_x + R_6 (I_2 - I_3) - V_B = 0$$

$$I_2 - I_1 = I_A$$

Mesh 3:

$$R_4 (I_3 - I_2) + R_5 (I_3 - I_4) + R_8 I_3 + R_9 I_3 + R_6 (I_3 - I_2) - \alpha i_x = 0$$

Mesh 4:

$$R_5 (I_4 - I_3) - V_A + R_7 (I_4 - I_5) = 0$$

Mesh 5:

$$I_5 = I_B$$

$$i_x = I_1$$

$$1) (R_2 + R_3 + \alpha) I_1 + (R_4 + R_6) I_2 - (R_4 + R_6) I_3 = V_B - V_A$$

$$2) -I_1 + I_2 = I_A$$

$$3) -\alpha I_1 - (R_4 + R_6) I_2 + (R_4 + R_5 + R_6 + R_8 + R_9) I_3 - R_5 I_4 = 0$$

$$4) -R_5 I_3 + (R_5 + R_7) I_4 - R_7 I_5 = V_A$$

$$5) I_5 = I_B$$

Problem 4:

a)

$$R_L = R_{TH}$$

$$R_{TH} = 7 \parallel 1 = 7/8$$

b)

When  $R_L$  is fixed,  $R_s = R_{TH}$  should be minimized to maximize power.

$$R_{TH} = (6 + R_o) \parallel 1 = (6 + R_o) / (7 + R_o)$$

$R_{TH}$  is minimized when  $R_o = 0$

c)

$$V_{TH} = V_{oc}$$

Using mesh analysis,  $V_{oc} = 3\text{mV}$

$$R_{TH} = 7/8$$

