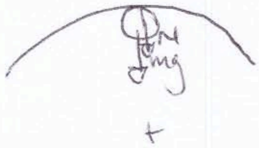
PROBLEM 1: SOLUTION

(a)



$$\Sigma F_y = N + mg = \frac{mv^2}{R}$$

"lose contact" $\Rightarrow N = 0$

$$mg = \frac{mv^2}{R}$$

$$v^2 = Rg$$

$E_i = mgh$ $\quad E_f = mg2R + \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\omega^2$

"rolling without slipping" $\Rightarrow \omega = \frac{V_{cm}}{r}$

$$E_f = mg2R + \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\frac{V_{cm}^2}{r^2}$$

$$I = \frac{2}{5}mr^2$$

$$E_f = mg2R + \frac{1}{2}mV_{cm}^2 + \frac{1}{5}mV_{cm}^2$$

$$E_f = mg2R + \frac{7}{10}mV_{cm}^2$$

$$E_i = E_f$$

$$\cancel{m}gh = \cancel{m}g2R + \frac{7}{10}\cancel{m}V_{cm}^2$$

$$h = 2R + \frac{7}{10}\frac{V_{cm}^2}{g}$$

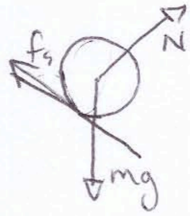
[$V_{cm}^2 = Rg$ from above]

$$h = 2R + \frac{7}{10}\frac{R\cancel{g}}{\cancel{g}}$$

$$h = 2R + \frac{7}{10}R$$

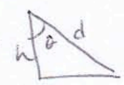
$$\boxed{h = \frac{27}{10}R}$$

(b)



$\boxed{W_{f_s} = 0}$ static* $(d=0)$

$\boxed{W_N = \cancel{N}d\cos 90} = \boxed{0}$

$\boxed{W_{mg} = mgh} = mg\,d\cos\theta$  $(d\cos\theta = h)$

* rolling without slipping means the surface of the sphere doesn't move relative to the surface of the ramp when touching. So the distance over which f_s acts is <u>zero</u>.

Problem 2. (Prof. Lin's 2nd Midterm)

(a) Since the starship 1 is orbiting along a circular path, the equation of motion in radial direction becomes

$$(F_{net})_{radial} = -\frac{GM_s m_1}{R^2} = -\frac{m_1 v^2}{R} \tag{1}$$

where M_s is the mass of the star and m_1 is the mass of starship 1. Thus, the mass of the star M_s can be expressed in terms of G, R and v,

$$M_s = \frac{vR^2}{G}. \tag{2}$$

(b) The speed of starship at the tangent point, v_2, is

$$v_2 = v + 0.5v = 1.5v \tag{3}$$

(note that $0.5v$ given in the question is the RELATIVE SPEED of starship 2 compared to starship 1). The escape speed for the starship 2 necessary for escaping the gravitational attraction by the star at the tangent point becomes

$$v_{esc} = \sqrt{\frac{2GM_s}{R}} = \sqrt{2}v \tag{4}$$

and we see that the starship 2 has a larger speed than the escape speed,

$$v_2 = 1.5v > v_{esc} = \sqrt{2}v \tag{5}$$

which implies that the starship can reach infinity or it has an unbounded orbit.

• Alternative solution: The cost one has to pay to bring starship 2 from the tangent point to the infinity will be

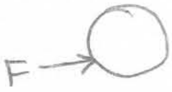
$$\Delta U = \left(-\frac{GM_s m_2}{r}\right)_{r\to\infty} - \left(-\frac{GM_s m_2}{r}\right)_{r=R} = \frac{GM_s m_2}{R} \tag{6}$$

while the kinetic energy of starship 2 at the tangent point is

$$\frac{1}{2}m_2(1.5v)^2 = \frac{9GM_s m_2}{8R} \tag{7}$$

and therefore, the starship 2 has enough kinetic energy to overcome the gravity of the star and reach the infinity. The starship 2 must have come from outside of the star system.

3 a) $J = F\Delta t$



$\Delta p = J = F\Delta t$

$mv = J$

$v = J/m \rightarrow$

$\Sigma\tau = F(.4r) = I\alpha = .4mr^2\alpha$

$\alpha = F/mr$ ↺ : const

$\omega = \alpha\Delta t = \frac{F\Delta t}{mr} = \frac{J}{mr}$ ↺

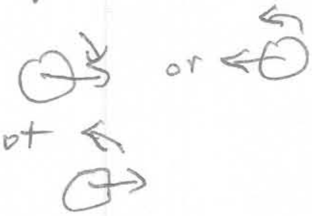
b) ω : No transfer : No F_f between B. Balls.

v : completely transfered ! Elastic $m_1 = m_2$.

$\omega_{1f} = \frac{J}{mr}$ ↺ $\omega_{2f} = 0$

$V_{1f} = 0$ $\quad V_{2f} = \frac{J}{m} \rightarrow$

c) ω and v of both balls will chang until $v = \omega r$

Note  or

but not

$\xrightarrow{+}$

Ball 1: 

$F_f = ma$

$V(t) = -\frac{F_f}{m}t$

↻+

$F_f r = I\alpha$

$\omega(t) = \frac{F_f r}{I}t - \frac{J}{mr}$

$v(t) = \omega(t)r \Rightarrow -\frac{F_f t}{m} = \frac{F_f r^2}{I}t - \frac{J}{m} = \frac{5}{2}\frac{F_f t}{m} - \frac{J}{m}$

$\frac{J}{m} = \frac{7}{2}\frac{F_f t}{m} \Rightarrow t = \frac{2}{7}\frac{J}{\mu_k mg}$

$V_f = -\frac{2}{7}\frac{J}{m}$ $\quad \omega_f = \frac{5}{7}\frac{J}{mr} - \frac{J}{mr} = -\frac{2}{7}\frac{J}{mr}$

Ball 2 : 

$V(t) = \frac{J}{m} - \frac{F_f t}{m}$ $\quad \omega(t) = \frac{F_f r t}{I}$

$V(t) = \omega(t)r \Rightarrow \frac{J}{m} - \frac{F_f t}{m} = \frac{F_f r^2 t}{I}$

$\frac{J}{m} = \frac{7}{2}\frac{F_f t}{m}$

$t = \frac{2}{7}\frac{J}{\mu_k mg}$

$V_f = \frac{5}{7}\frac{J}{m}$ $\quad \omega_f = \frac{5}{7}\frac{J}{mr}$

d) No: F_f does work from slipping → No slipping.

Ball 1 : $E_i = \frac{1}{2}I\omega_i^2 = \frac{1}{5}\frac{J^2}{m}$ $\quad E_f = \frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2 = \frac{2}{35}\frac{J^2}{m} < E_i$

7A Midterm II - Lin
Problem 4 Solutions

Part A

Assume a rocket of mass $M + \Delta m$ is moving with a speed v. During some time, Δt, a mass Δm is ejected out the back with a velocity $-v_r$ (1000 m/s) with respect to the rocket ($v - v_r$ relative to an outside observer). As a result of this ejected mass, the rocket (now only of mass, M) is moving slightly faster, $v + \Delta v$.

Because this is a sort of inelastic collision (energy is lost in the burning of the fuel), we can investigate the problem using conservation of momentum. The initial momentum (defining the direction of motion of the rocket to be positive) of the system is,

$$p_i = (M + \Delta m)v$$

And the final momentum is,

$$p_f = \Delta m(v - v_r) + M(v + \Delta v)$$

So expanding these expressions and setting them equal to each other, we have that

$$Mv + \Delta mv = \Delta mv - \Delta mv_r + Mv + M\Delta v$$

We can cancel both terms on the right hand side, then and we have:

$$0 = -\Delta mv_r + M\Delta v$$

or,

$$\Delta mv_r = M\Delta v$$

Now, because this takes some time, Δt, to happen, we can divide by Δt on both sides of the previous equation, leaving us with,

$$\frac{\Delta m}{\Delta t}v_r = M\frac{\Delta v}{\Delta t}$$

Now we have some familiar terms here. The first, $\frac{\Delta m}{\Delta t}$, is the rate at which the gas cloud behind the rocket is gaining mass. This is equal to the rate at which the rocket is losing mass. So we can write that the current mass of the rocket is

$$M(t) = M_0 - Rt$$

where M_0 is the original mass of the rocket (5500 kg), R is the mass loss rate ($\frac{\Delta m}{\Delta t} = R = 100$ kg/s).

We also have $\frac{\Delta v}{\Delta t}$, which is the acceleration of the rocket. So we have:

$$Ma = Rv_r$$

Where we recognize Ma as the total force on an object. So the thrust on the rocket is:

$$F_{thrust} = Rv_r$$

Then the acceleration of the rocket is:

$$a = \frac{Rv_r}{M}$$

We can replace the mass, M, on the right hand side with $M(t)$, the mass of the rocket at any time, and get:

$$a = \frac{Rv_r}{M(t)}$$

So at time $t = 0$, we are given that the rocket has a mass of 5500 kg. This gives an acceleration of

$$a = \frac{100kg/s \times 1000m/s}{5500kg} = 18.2m/s^2.$$

At the time of burnout, the rocket has reduced its mass by 80%, so the mass at burnout is 1000 kg($0.20 \times 5500kg = 1100kg$). This gives an acceleration of:

$$a = \frac{100kg/s \times 1000m/s}{1100kg} = 90.9m/s^2.$$

Now, substituting in the full form of $M(t)$, we have:

$$a = \frac{Rv_r}{M_0 - Rt}$$

Integrating with respect to time gives us:

$$\begin{aligned}
\int_{t_0}^{t_f} a\,dt &= \int_{t_0}^{t_f} \frac{Rv_r}{M_0 - Rt}\,dt \\
v(t_f) - v(t_0) &= -v_r \ln(M_0 - Rt)\Big|_{t_f}^{t_0} \\
v(t_f) &= -v_r \ln M(t)\Big|_{t_f}^{t_0} \\
&= -v_r(\ln M(t_f) - \ln M(t_0)) \\
&= -v_r \ln\left(\frac{M(t_f)}{M(t_0)}\right) \\
&= v_r \ln\left(\frac{M(t_0)}{M(t_f)}\right)
\end{aligned}$$

Now, in this case, Mt_f, is twenty percent of $M(t_0)$, so their ratio is 5:

$$v_f = v_r \ln 5 = 1000m/s * \ln 5 = 1609m/s$$

Part B

This problem can be done two ways: both involve conservation of momentum and one of of which uses the center of mass.

Center of Mass Method

We are going to measure everything from position of the spacestation. The spaceship has an unknown mass distribution, so we can just say that its center of mass located a distance, d, away from the station and its total mass is M. We also know that the astronaut is located 1m from the station with a mass, m. When she pushes off, she will cause the rocket to move in the opposite direction. The initial momentum in this reference frame is zero, so the center of mass does not move. So we can calculate how far the ship can possibly move.

Initially, the center of mass of the astronaut-rocket system is located at

$$\bar{x} = Md + m(1m)$$

After the astronaut has hit the back of the rocket, she has traveled a distance of the length of the rocket, minus however far the rocket has moved, s, so her new position is $(1m + l - s)$. The rocket's new position is then $(d - s)$. So the center of mass afterwards is located at

$$\bar{x} = M(d - s) + m(1m + l - s)$$

Because the center of mass cannot move, we know that these positions are equal:

$$Md + m(1m) = M(d - s) + m(1m + l - s) = Md - Ms + m(1m) + ml - ms$$

Simplifying, this leaves:

$$0 = -Ms + ml - ms$$

Solving for s, the distance the ship has moved,

$$\begin{aligned} s &= \frac{ml}{M + m} \\ &= \frac{100kg \times 11m}{100kg + 1000kg} \\ &= 1m \end{aligned}$$

So the ship *barely* makes it.

Momentum Conservation and Kinematics

Another method for this problem actually uses the information about the astronauts speed that was given. We again have to use momentum conservation, but in a more transparent way (instead of just saying that it preserved the location of the center of mass).

The astronaut pushes off the forward wall of the spaceship with a speed, $v = 10m/s$, causing the spaceship to recoil at a speed v_s, related by momentum conservation:

$$mv - Mv_s = 0$$

or,

$$v_s = -\frac{m}{M}v = \frac{100kg}{1000kg}10m/s = -1m/s$$

So the relative velocity between the spaceship and the astronaut is $v - v_s$. Because the astronaut travels a distance, $l = 11m$ relative to the spaceship, the time this takes is:

$$\begin{aligned} t &= \frac{l}{v - v_s} \\ &= \frac{11m}{10m/s + 1m/s} \\ &= 1s \end{aligned}$$

The spaceship is moving at a speed $1m/s$ towards the space station for $1s$, going $1m$. Again, we see that the rocket barely gets there.

Problem 5 Solution

Note: Define right as the positive x-direction and up as the positive y-direction.

1 Part (a)

The final kinetic energy of the system cannot be greater than the initial kinetic energy of the system otherwise we'd be creating energy in the system. Since we have $\vec{v}_1 = -\vec{v}_0$ we know their magnitudes are the same, $v_1 = v_0$.

$$\begin{aligned} KE_i &\geq KE_f \\ \frac{1}{2}mv_0^2 &\geq \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 \\ 0 &\geq \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 \end{aligned}$$

Since the right hand side can never be negative, we see that $v_2 = v_3 = 0$ and the balls 2 and 3 must remain at rest. It is clearly not possible for these balls to be completely unaffected by the collision. Quantitatively, we note that momentum must be conserved, and given the above condition $mv_0\hat{x} \neq -mv_0\hat{x}$. Momentum would not be conserved.

$\Longrightarrow$ **No**, it is **not possible** for ball 1 to bounce backwards with the same speed.

2 Part (b)

Using momentum conservation in the x and y directions we have

$$\begin{aligned} p_x : mv_0 &= mv_{2,x} + mv_{3,x} \\ v_0 &= v_{2,x} + v_{3,x} \\ p_y : 0 &= mv_{2,y} - mv_{3,y} \\ v_{2,y} &= v_{3,y} \end{aligned}$$

The only constraint on the x-components of v_2 and v_3 is that they add up to v_0 and they need not be equal. The y-components of v_2 and v_3 must always be equal. Because of this, the angles can be different.

$\Longrightarrow$ **Yes**, it **is possible** for the angles to be different.

3 Part (c)

Again, using momentum conservation and that $|v_1| = |v_2| = |v_3| = v$

$$\begin{aligned} p_y : 0 &= mv_2 \sin\theta_2 - mv_3 \sin\theta_3 \\ 0 &= v\sin\theta_2 - v\sin\theta_3 \\ \sin\theta_2 &= \sin\theta_3 \\ \theta_2 &= \theta_3 = \theta \\ p_x : mv_0 &= -mv_1 + mv_2\cos\theta_2 + mv_3\cos\theta_3 \\ v_0 &= -v + v\cos\theta + v\cos\theta \\ v_0 &= (2\cos\theta - 1)v \end{aligned}$$

The only constraint is that the term $(2\cos\theta - 1)$ must be positive since v is positive. We can see there are many angles for θ that will fulfill this condition.

$\Longrightarrow$ **Yes**, it **is possible** for them all to have the same speed but different directions.

Problem 6 LIN

Part A)

Conservation of momentun:

$$m_2 v = (m_1 + m_2) v_2$$

$$v_2 = \frac{m_2 v}{m_1 + m_2}$$

After collision, Use Conservation of Energy:

$$\frac{1}{2}(m_1 + m_2){v_2}^2 = \frac{1}{2}kd^2$$

Use v2 from above and solve for d

$$d = \frac{m_2 v}{\sqrt{k(m_1 + m_2)}}$$

PartB)
Energy initial = Energy final + Energy Lost

$$Energy_{Lost} = \frac{1}{2}m_2 v^2 - \frac{1}{2}kd^2$$

$$\frac{Energy_{Lost}}{InitialKE} = 1 - \frac{m_2}{m_1 + m_2} = \frac{m_1}{m_1 + m_2}$$

PartC)

$$\bar{F}_{avg} = \frac{\Delta p}{\Delta t} = \frac{m_2 \Delta v}{\Delta t}$$

$$\Delta v = \frac{m_2 v}{m_2 + m_1} - v$$

$$F_{avg} = \frac{-m_1 m_2 v}{\Delta t(m_1 + m_2)}$$

Note: Just magnitude accepted as well.