Midterm 2

October 30^{th}, 2012
120minutes ⋄ 100points

Physics 7A
University of California at Berkeley

This midterm is closed book and closed notes. You are allowed a double sided paper on which you may write whatever you wish. You are not allowed to use calculators. Anyone who does use a wireless capable device will automatically receive a zero for this midterm. Cell phones must be turned off.

Please make sure that you do the following during the midterm:

- ⋄ Write your name, discussion number, ID number on all documents you hand in.
- ⋄ Make sure that the grader knows what s/he should grade by circling your final answer.
- ⋄ Cross out any parts of your solutions that you do not want the grader to grade.
- ⋄ Answer all questions that require a numerical answer to one significant figure.
- ⋄ For questions without numerical values give all your answers in terms of variable names specified in the text of the question and well known physical constants.
- ⋄ Your answers might not depend on all given values/variables.

We will give partial credit on this midterm, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems. If at any point in the exam you have any questions, just raise your hand, and we will see if we are able to answer them.

Problem 1 ⋄ *20points* ⋄ Monkey Roller coaster

Wondering what's new in monkey-land?! Monkeys are designing an awesome roller coaster ride that has a track like in Figure 1 and carts that are balls that can roll on these tracks. The cart with a monkey in it has mass m, radius r and you can assume uniform mass density so that the moment of inertia of a cart about its center is $I = \frac{2}{5}mr^2$.

A cart is released from rest at the top of the ramp of height H. It rolls without slipping down the ramp and then inside a loop and over a bump. Assume that the coefficient of friction between the cart and the track is sufficiently large that the cart rolls without slipping the entire time. All distance in this problem (H, $R_{\mathbf{A}}$, $R_{\mathbf{B}}$, and h) are much larger than r.

- ⋄ **A** Find the largest radius $R_{\mathbf{A}}$ of the loop such that cart barely stays in contact with the track at its highest point.
- ⋄ **B** What is the smallest radius $R_{\mathbf{B}}$ of a semicircular bump so cart stays in contact with the track on top of it?

After it passes the bump the cart moves up a ramp that is completely frictionless so that the cart is slipping.

- ⋄ **C** To what height h will the ball go up the frictionless ramp?

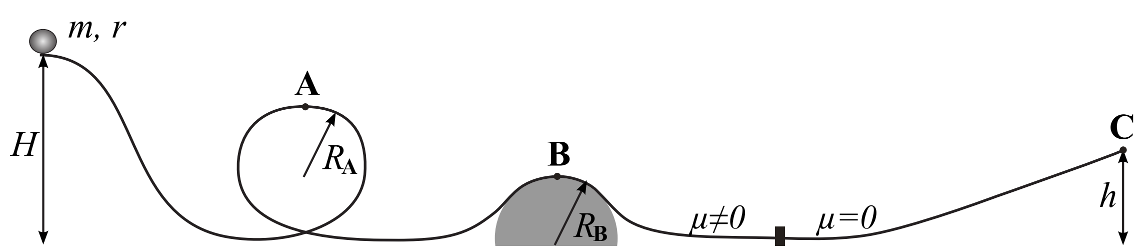


Figure 1: Problem 1: Monkey Roller Coaster

Problem 2 ⋄ *25points* ⋄ Monkey Rocket

More ambitious monkeys are working on a rocket-ship which is their first and will not go very far so you can model it as going vertically upward in a constant gravitational field.

The rocket is made of an engine with mass m_E and fuel that initially has mass m_F. The fuel is burned at a constant rate κ and then expelled downward at speed u relative to the rocket. The rocket-ship is at rest at blastoff.

- ⋄ **A** What is the speed of this rocket as a function of time before burnout time?
- ⋄ **B** What is the speed of the rocket at burnout?
- ⋄ **C** What is its height at this time?

Due to budget cuts monkeys have to make a much simpler rocket that expels all its fuel at the initial instant with the same relative speed u.

- ⋄ **D** What is the speed of this rocket at the burnout time of the rocket in parts **A** and **B**?
- ⋄ **E** What is its height at this time?

PROBLEM 3 ⋄ *25points* ⋄ MONKEY ON A ROD

A sticky monkey of mass m is moving with speed ν toward a rod of length L and mass M laying on a frictionless horizontal surface in the x-y plane. The velocity of the monkey is perpendicular to the rod so that it hits the rod a distance D ($D < L/2$) away from its center and sticks to it. Describe the subsequent motion of the rod both qualitatively and quantitatively.

Please use the coordinate system as in Figure 2a with the origin at the center of the rod before it starts moving. The sticky monkey is much smaller than the rod and can be modeled as a point particle. Moment of inertia of the rod about its center is $I = \frac{1}{12}ML^2$.

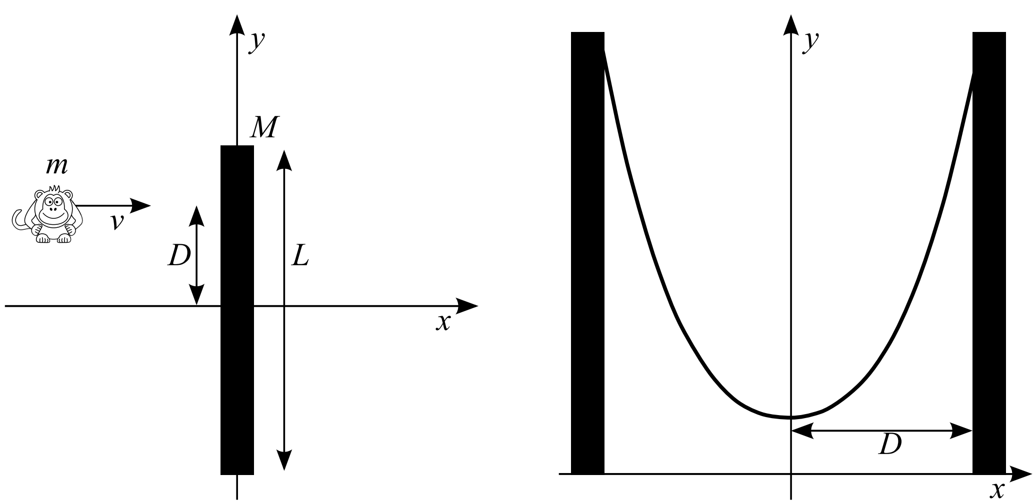
PROBLEM 4 ⋄ *30points* ⋄ MONKEYS' BRIDGE

- ⋄ **A** Derive a differential equation for the shape of a hanging rope suspended from equal height between two poles like in Figure 2b. The rope has uniform weight per unit length w and total length $2L$. The tension in the middle of the rope is T_0 when the distance between the poles is $2D$ ($D < L$).
- ⋄ **B** You do not have to integrate the equation from part **A** — the resulting curve is $y(x) = a\cosh(\frac{x}{a})$ and it is called catenary shape. What is a in terms of physical variables w and T_0 or L and D?
- ⋄ **C** What is the angle that the tension force makes with the horizontal at the ends of the rope?
- ⋄ **D** Use this angle to write an equation for T_0 in terms of w, L, and D. (You do not need to solve it.)

You might find these hyperbolic identities useful:

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \frac{d\sinh(x)}{dx} = \cosh(x)$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \frac{d\cosh(x)}{dx} = \sinh(x) \qquad \cosh^2(x) - \sinh^2(x) = 1.$$



(a) PROBLEM 3: MONKEY ON A ROD (b) PROBLEM 4: MONKEYS' BRIDGE

Figure 2