Problem 1. [20 points]

Your room on the lower deck of cruise liner is below sea level and is heated to a comfortable 25°C. The water outside is 5°C. One of your walls is the hull of the ship. The hull is 5cm steel with a thermal coefficient of 15 W/m-°C. The wall area is 12m^2. You can ignore the ceiling, floor and other three walls of your room in this problem.

(a) What is the rate of heat transfer through this wall?

Solution:

Since there is no sink or source through this wall, the rate of heat transfer is a constant.

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} = Const \rightarrow \left(\frac{dQ}{dt}\right)\int_0^{d_{wall}} dx = -kA\int_{T_{room}}^{T_{water}} dT$$

Therefore,

$$\frac{dQ}{dt} = -kA\frac{T_{water} - T_{room}}{d_{wall}} = 72\,kW$$

(b) Your heater is a heat pump operating at its ideal Carnot performance. How much power must be supplied to the heater?

Solution:

The assumed ideal Carnot performance indicates that

$$\eta = \frac{\dot{Q}_{room}}{\dot{W}} = \frac{T_{room}}{T_{room} - T_{water}}, \text{ i.e.}, \dot{W} = \frac{dQ_{room}}{dt}\left(1 - \frac{T_{water}}{T_{room}}\right)$$

Where $\dot{Q}_{room}$ stands for the heat released per unit time to the room, which possesses a higher temperature. In order to keep the temperature of the room constant, we require

$$\frac{dQ_{room}}{dt} = \frac{dQ}{dt}$$

Therefore,

$$\dot{W} = \frac{dQ}{dt}\left(1 - \frac{T_{water}}{T_{room}}\right) = 72\text{kW} \times \left(1 - \frac{278}{298}\right) = 4.83\,kW$$

Problem 2. **[20 points]**

A hot rock of mass m=10 kg at a temperature T=1500 K is placed into a container which is filled with $1m^3$ of water at a temperature of 300 K. The specific heat of water can be approximated as 4kJ/kg-K and the specific heat of the rock is 1kJ/kg-K. The density of water is $1g/cm^3$.

(a) What is the final temperature of the rock and water?
(b) What is the entropy change of the rock?
(c) What is the entropy change of the water?
(d) By how much did the height of the water change due to heating alone (ignoring the bigger effect of the addition of the rock) if the container has a depth of 2m and a surface area of $0.5m^2$? The coefficient of volume expansion of water is $2x10^{-6}$/°C.

Solutions:

(a) Assume that the final temperature is T_f.

$$c_R m(T_f - T) + c_w m_w (T_f - T_w) = 0$$

$$T_f = \frac{c_R mT + c_w m_w T_w}{c_R m + c_w m_w} = 303\,K$$

(b) $\Delta S_{rock} = \int_T^{T_f} \frac{c_R m d T'}{T'} = c_R m ln\left(\frac{T_f}{T}\right) = 10\ln\left(\frac{303}{1500}\right) kJ/K = -16\,kT/K$

(c) $\Delta S_{water} = \int_{T_w}^{T_f} \frac{c_w m_w dT}{T} = c_w m_w \ln\left(\frac{T_f}{T_w}\right) = 4\times10^3 \ln\left(\frac{303}{300}\right) kJ/K = 40\,kT/K$

(d) The change of the volume of the water according to the thermal expansion is

$$\Delta V = \beta V (T_f - T_w) = 6\times10^{-6} m^3$$

Therefore, the change of the height is

$$\Delta h = \frac{\Delta V}{A} = \frac{6\times10^{-6} m^3}{0.5\,m^2} = 1.2\times10^{-5} m$$

Problem 3. [14 points]

A thunderstorm gets energy by condensation of water vapor in the air. Suppose that the storm condenses all the water vapor in $10km^3$ (ten cubic kilometers) of air. Assume that at the conditions of the storm there is 100% relative humidity and that this corresponds to .02 kg of water in every cubic meter and a heat of vaporization of $2x10^3$kJ/kg.

(a) How much heat is released by the storm through this condensation process?
(b) How many kilotons of TNT does this correspond to if 1kiloton of TNT releases $4x10^{12}$J.

Solutions:

(a) $Q=\rho_w V_{air} L_V=0.02\,kg/m^3 \times 10\,km^3 \times 2 \times 10^3\,kJ/kg=4 \times 10^{11}\,kJ$

(b) $m_{TNT}=\dfrac{4 \times 10^{11}\,kJ}{4 \times 10^{12}\,J/kiloton}=100\,kiloton$

Problem 4. [32 points] A heat engine uses n moles of an ideal gas as its working substance. The ideal gas has a ratio of specific heats $\frac{c_p}{c_v} = \gamma$. The quantity γ should be left as a symbol anywhere it appears in the answers to the questions below.

The engine has the following three-step reversible cycle. It starts at State 1 where the gas has temperature T_1 and volume V_1. The steps are:

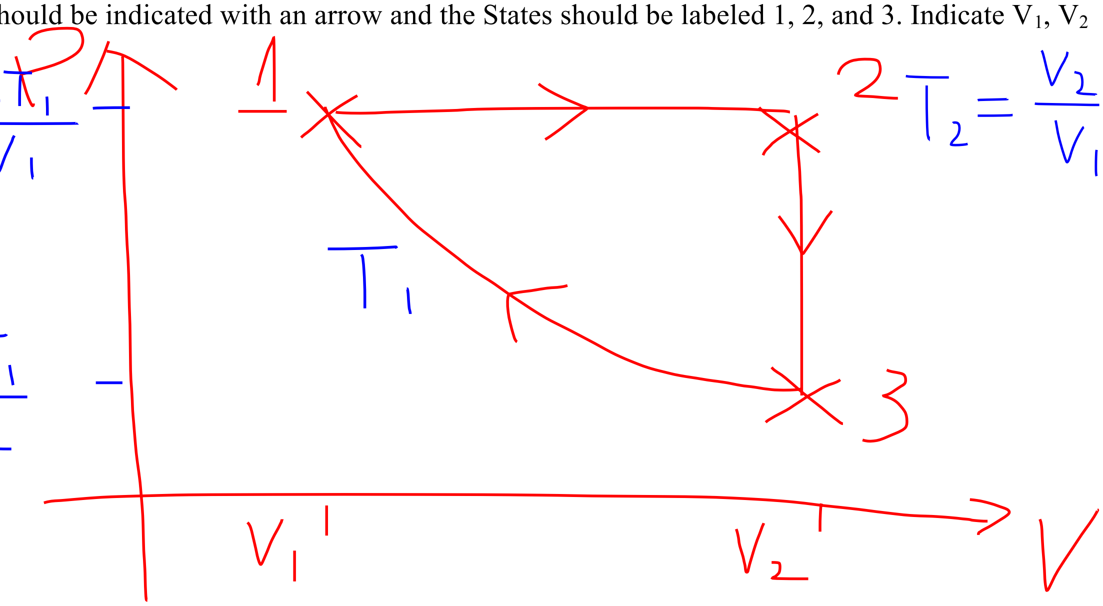
1. From State 1 to State 2: An expansion at constant pressure from V_1 to V_2
2. From State 2 to State 3: A decrease in pressure at constant volume V_2
3. From State 3 to State 1: An isothermal compression to volume V_1

(a) [8 points] Sketch this cycle in a P-V diagram. The sketch does not need to be to scale. The direction of each step should be indicated with an arrow and the States should be labeled 1, 2, and 3. Indicate V_1, V_2 on the plot.

$$P_2 = P_1 = \frac{nRT_1}{V_1}$$

$$T_2 = \frac{V_2}{V_1} T_1$$

$$P_3 = \frac{nRT_1}{V_2}$$



1@ for state 1, 2 and 3
1@ for direction of arrows
1@ for Volumes

b) [20 points] Find Q, W and ΔS at States 1,2 and 3, and fill in the table below. Do not use the fact that the sum of ΔS must be zero when you calculate ΔS.

	ΔU	W	Q	ΔS
$1\to 2$	~~1~~ $nC_v\left(\frac{V_2}{V_1}-1\right)T_1$	$nRT_1\left(\frac{V_2}{V_1}-1\right)$	$nC_pT_1\left(\frac{V_2}{V_1}-1\right)$	$nC_p\ln\frac{V_2}{V_1}$
$2\to 3$	~~2~~ $nC_v\left(1-\frac{V_2}{V_1}\right)T_1$	0	$-nC_vT_1\left(\frac{V_2}{V_1}-1\right)$	$-nC_v\ln\frac{V_2}{V_1}$
$3\to 1$	~~3~~ 0	$-nRT_1\ln\frac{V_2}{V_1}$	$nRT_1\ln\frac{V_2}{V_1}$	$-nR\ln\frac{V_2}{V_1}$

+1 if tried
1@ for correct expression
1@ for expressing in terms of the knowns
+1 if perfect

$\Delta U = nC_v\Delta T$

$W=\int P\,dV = P\Delta V$ for isobaric

$= nRT\ln\frac{V_f}{V_i}$ for isothermal

$Q=\Delta U + W$

$Q = nC_v\Delta T$ for isochoric

$Q = nC_p\Delta T$ for isobaric

$\Rightarrow \Delta S = \begin{cases} nC_v\ln\frac{T_f}{T_i} \\ nC_p\ln\frac{T_f}{T_i} \end{cases}$

isothermal: $\Delta S = \frac{Q}{T}$

Or use $\Delta S = nC_v\ln\frac{T_f}{T_i} + nR\ln\frac{V_f}{V_i}$

as shown in discussion session

(c) [4 points] Verify that ΔS summed over all three steps is zero.

$\Delta S_{total} = nC_p\ln\frac{V_2}{V_1} - nC_v\ln\frac{V_2}{V_1} - nR\ln\frac{V_2}{V_1}$

$=0$ as $C_p = C_v + R$ //

1 for giving a try
2 for correct expression
3 for assuming a certain kind of gas to evaluate
4 for perfect

7@ part
2 for giving it a try
4 for getting in the right direction
7 for perfect

Problem 5. [14 points]

(a) A physicist is studying a sample monatomic gas in a container (the volume of the container is V). The mean free path of an atom is 10^{-9} m when the gas is at T=400°K and at atmospheric pressure.

What is the mean free path if the volume of the container is reduced to V/2? Assume the temperature and number of particles is held fixed.

(b) The surface of an object has area A=10m^2 is at 300°K emissivity of 0.8. What happens to the object if it is in a dark room with walls at 200°K? Write an equation for what you expect will happen; you do not need to solve it. You can use a symbol for any physical constants you need so long as you define them. In particular, the Stephan-Boltzmann constant is σ_{SB}. You do not need to evaluate anything numerically.

-1 if compute 10^9/2 incorrectly

a) $$l_M = \frac{1}{4\pi\sqrt{2}\, r^2 \frac{N}{V}}$$

$$l_M' = \frac{1}{4\pi\sqrt{2}\, r^2 \frac{N}{V/2}} = \frac{1}{2} l_M$$

i.e. the mean free path is reduced by a half: 5×10^{-10} m //

b) In this question it can get very complicated by writing down all the parameter that one can write down including the heat capacity, surface area of the wall, etc. and solve the whole thing in time.

However the simplest way to put it is to assume the wall is much larger (i.e. has much higher heat capacity) and we wait long enough for equalibrium.

As the wall is enclosing the ojbect, at equalibrium

T=200K

If one do not assume the wall is much higher, you would get an equalibrium between 200-300K.

One student even consider the wall rediating all energy to the universe. In this case the equalibrium will be at 0K.

So the answer leads to perfect score would be: 1) will reach equalibrium 2) at a certain temperature in one of the above 3 cases. Anything more than this would receive full marks (e.g. Writing down the differential equations in time, or even