Midterm 1

12:40- 2:00, February 19

Notes: There are **five** *questions on this midterm. Answer each question part in the space below it, using the back of the sheet to continue your answer if necessary. You can use any facts in the lecture notes without deriving them again.* **None of the questions requires a very long answer, so avoid writing too much! Unclear or long-winded solutions may be penalized.** *The approximate credit for each question part is shown in the margin (total 50 points).*

Your Name: **Your Lab Section:**

Name of Student on Your Left:
Name of Student on Your Right:

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Q1	
Q2	
Q3	
Q4	
Q5	
Total	

1. (14 points) Short questions:

   a) (1 point) If I am talking about something that can be everything, then I am talking about:

      i) signals
      ii) systems
      iii) signals and systems
      iv) something
      v) nothing

   b) (2 points) Consider the system:

$$y(t) = 2x(t^2).$$

      Is the system linear?

c) (3 points) Consider the Foucault pendulum system discussed in lecture. The input $x(t)$ to the system is the applied force and the output of the system $y(t)$ is the angle the string makes with the vertical. They are related by the differential equation:

$$mr\frac{d^2y}{dt^2} + mg\sin y(t) = x(t).$$

Suppose the applied force $x(t)$ is a sinusoid at frequency 100 Hz before time $t = 2$s and becomes a sinusoid at frequency 200 Hz after time $t = 2$s. Is the system time-invariant? Explain.

d) (4 points) A continuous-time speech signal has no frequency components beyond 20 kHz. It is sampled at the rate of $30,000$ samples per second. What is the highest non-zero frequency component (in Hz) of the continuous-time signal captured in the discrete-time samples? What about if the sampling rate is $50,000$ samples per second?

e) (4 points) A periodic input signal of period p is fed into a LTI system. Is the output necessarily periodic? If not, give a counter-example. If so, give an argument to explain why and compute the period of the output signal.

2. (8 points) You have measured the frequency response of H of a (real) LTI system at angular frequency 100 rad/s, i.e. you know $H(100)$. Based on this information alone, give the output to the input signal $x(t)$ in each of the parts below, or explain why there is not enough information to compute it.

a) (2 points) $x(t) = \sin(100t + \frac{\pi}{4})$;

b) (2 points) $x(t) = \sin^2(100t)$;

c) (2 points) $x(t) = \sin(100t) + \cos(100t)$;

d) (2 points) $x(t) = e^{-i100t}$.

3. (7 points) Consider the following signal defined on the reals:

$$x(t) = \begin{cases} 1 & \text{if } t \in [n, n+0.5) \text{ for some even integer } n \text{ or } t \in [n, n+1) \text{ for some odd integer } n \\ -1 & \text{if } t \in [n+0.5, n+1) \text{ for some even integer } n \end{cases}$$

a) (2 points) Plot the signal $x(t)$ as a function of t, and label the key features.

b) (2 points) Is the signal periodic? If so, calculate the period. If not, explain.

c) (3 points) The signal passes through a system with input-output relationship given by :

$$y(t) = 2x(\frac{t}{2} - 1).$$

Plot the output signal $y(t)$ for the given input, and label the key features.

4. (6 points) Consider a discrete-time periodic signal $x(n)$ of period p. You can assume p is even.

   a) (2 points) Give the Fourier series expansion of $x(n)$ in terms of cosines.

   b) (4 points) From the expansion in part (a), derive the Fourier series expansion of $x(n)$ in terms of complex exponentials. Give the coefficients in terms of the coefficients of the expansion in part (a).

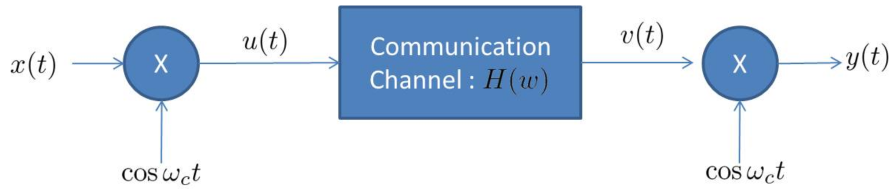

Figure 1: Diagram for the communication system

5. (15 points) A communication system is depicted in Figure 1. The communication channel is modeled by a LTI system with a frequency response given by:

$$H(\omega) = \begin{cases} A & \text{if } \omega_c - \frac{W}{2} \leq |\omega| \leq \omega_c + \frac{W}{2} \\ 0 & \text{else} \end{cases}$$

where $A > 0$. The frequency ω_c is called the *carrier frequency* and the parameter W is called the *bandwidth*. You can assume that $\omega_c \gg W$. For example, in cellular communication, the carrier frequency may be 1 GHz and the bandwidth may be 10 MHz.

a) (2 points) Plot the magnitude of the frequency response of the communication channel, labeling the key features.

b) (5 points) Suppose the input signal is a sinusoid $x(t) = \cos \omega t$ with $\omega \in (0, \frac{W}{2})$. Give explicit expressions for the signals $u(t), v(t)$ and $y(t)$. What do you think is the purpose of multiplying the input signal by $\cos \omega_c t$?

c) (3 points) Consider the overall system with $x(t)$ as the input and $y(t)$ as the output. Is the system LTI? Why or why not?

d) (5 points) Suppose you are allowed to further process $y(t)$ by passing it through a LTI system to yield another output $\tilde{y}(t)$. Design the frequency response of this LTI system such that $\tilde{y}(t) = x(t)$.