Wednesday, October 6, 9:30–11:00 AM, 1999.

Please write all answers in the space provided. If you need additional space, write on the back sides. Indicate your answer as clearly as possible for each question. *Write your name at the top of each page as indicated.*

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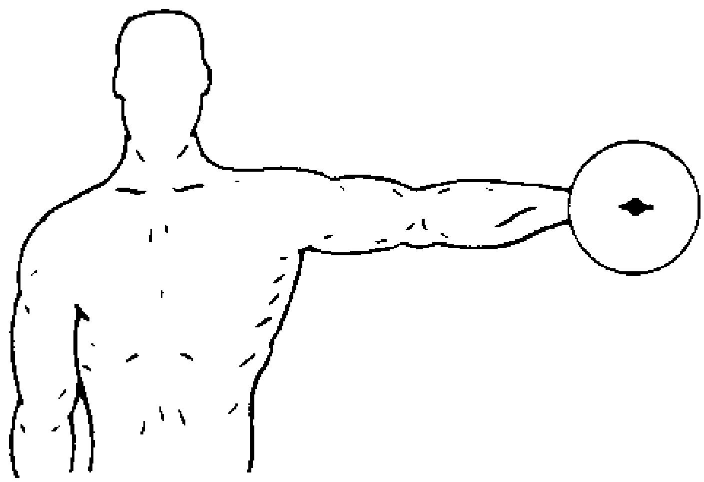
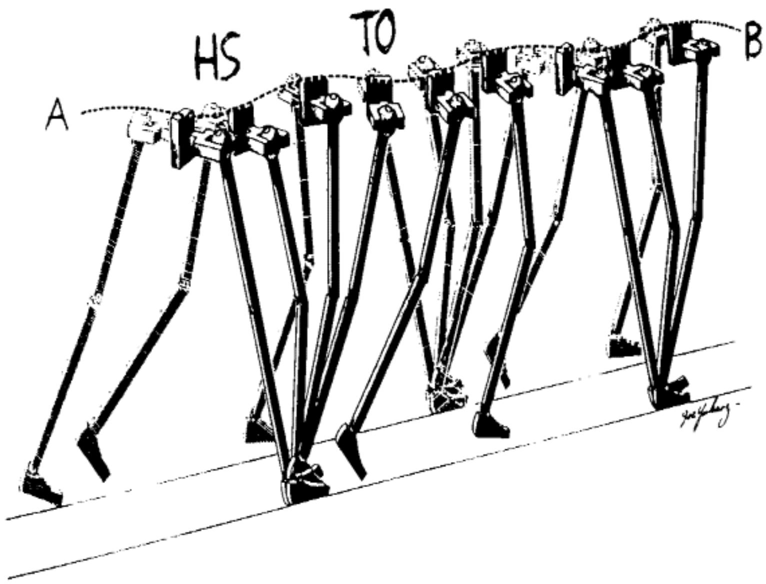
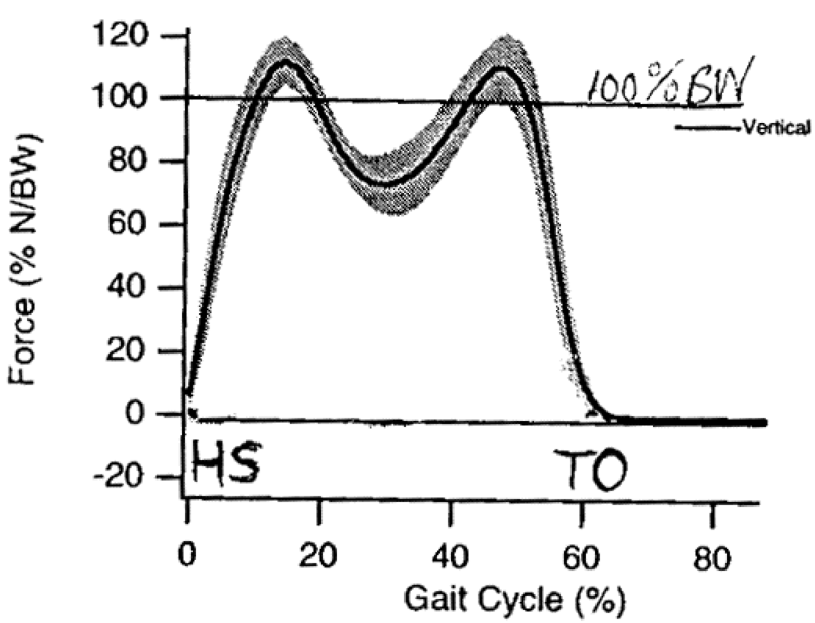
1. Statics (15 points)

Using a graphical approach (vector force triangle), estimate the magnitude of the *joint contact force* at the shoulder for the static abduction exercise shown schematically here (to scale). The mass of the weight in the hand is 6 kg. Ignore the weight of the arm and assume that only one muscle group is active. State any additional assumptions necessary to complete your analysis.



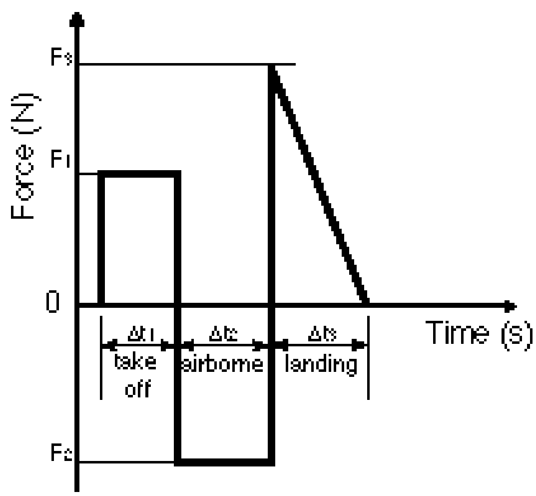
2. Dynamics and Gait (15 points)
On the top a model of human walking, with the dashed line A–B showing the approximate trajectory of the center of mass of the body during gait, and the letters HS and TO denoting heel strike and toe-off, respectively. Assume that A–B is sinusoidal, with minimum values at HS and TO (*i.e.* one full period of motion between these two points), and maximum value half way between these points. Using this assumption and the equations of motion, explain why the vertical component of the ground reaction force acting on the foot (shown on the bottom) varies as it does around one-times body-weight and is not simply constant (only one foot is making contact with the ground between just after HS and just before TO).





3. Dynamics and Impact (25 points)

A person stands at rest on a force platform. The force reading is then set to zero. Next, they jump up from the platform and land an instant later. The response of the platform, which measures the vertical force exerted by the person on the platform as a function of time, is idealized below. Assuming the person can be modeled as a single rigid body, calculate:
(i) the mass of the person;
(ii) their velocity at take-off;
(iii) the maximum height of their jump;
(iv) their velocity at landing.



F_1 = 600 N Δt_1 = 0.3 s

F_2 = -800 N Δt_2 = 0.4 s

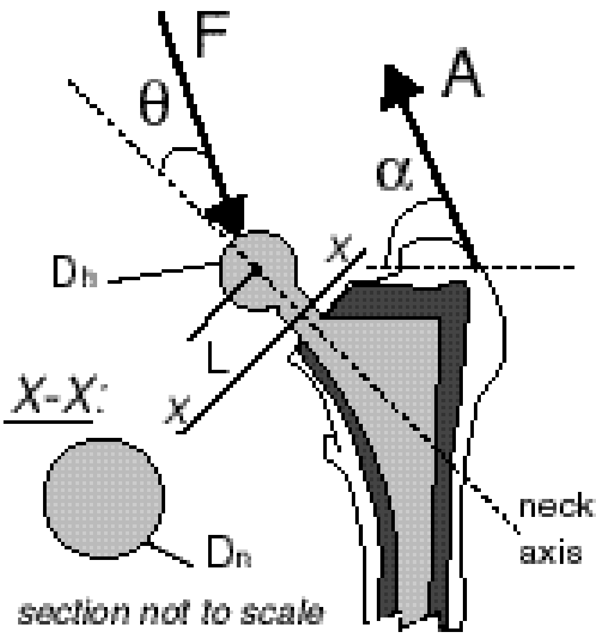
F_3 = 1000 N Δt_3 = 0.4 s

4. Implant Design (25 points)

a. Write out the expression for the maximum tensile stress at section X-X on the neck portion of the hip prosthesis shown below. (Note, the force **F** goes through the head center of the prosthesis).

b. What is the main geometric design variable for ensuring that the neck does not fracture, and why?

c. When testing candidate designs, we use forces typical of what would occur during gait, but apply them with θ=90°to simulate worst case conditions. For this situation, calculate a value for your main design variable (from part *b*) so that the maximum allowable stress equals the endurance limit (say about half of the static strength). In doing so, make an informed estimate of all required parameters.



Nomenclature

F = joint contact force vector at angle θ to neck axis

A = abductor muscle force vector at angle α to horizontal

D_n = diameter of prosthesis neck at section X-X

D_h = diameter of prosthesis head

L = distance from center of prosthesis head to section X-X

5. Composite Beam Theory (20 points)
For the cross-section shown, write out an expression for the stress at point *A* in terms of the dimensions *h* and *b*, the applied pure bending moment *M*, and the modulus *E*. Assume point A is bending in tension.