

Wednesday, October 10, 9:30–11:00 AM, 2001.

Please write all answers in the space provided. If you need additional space, write on the back sides. Indicate your answer as clearly as possible for each question. *Write your name at the top of each page as indicated.*

1. (20 points total) Forces and Moments at Joints

Fitness instructors often recommend doing different types of sit-ups to tone the abdominal muscles, although some exercises may be more strenuous on the lower spine. Two common types of sit-ups are the leg-raised sit-up and the crunch. In the first case, you start by lying fully flat on your back, legs straight and arms by your side. Then, you slowly raise your legs just slightly off the ground. In the second case (the crunch), you start from the same initial position, but instead of raising your legs you raise your shoulders and head off the ground, slowly and just slightly. The question is, *how do these different types of sit-ups affect the magnitude of the joint contact force on the L2 vertebral body?* Express your answer as the ratio of the joint contact force (in terms of body weight) acting on L2 when doing the leg-raise sit-up to the force on L2 during the crunch.

Use free-body diagrams to support your analysis, and state all assumptions.

2. (30 points total) Joint Dynamics

An incomplete free-body diagram of the leg and foot of a person running up stairs is shown above. The magnitude and direction of the ground reaction force, F , is shown in the diagram and is to scale. The direction of the force, P , in the patellar ligament is also shown but its magnitude is not known.

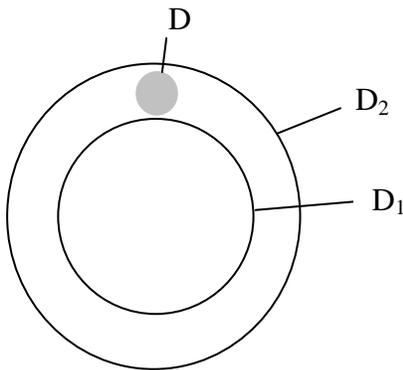
- (a) Complete the free-body diagram by showing the joint contact force, J . Show the direction of J as accurately as you can. For the purposes of doing this, ignore the mass of the leg.
- (b) In the space to the right of the figure, construct the corresponding vector force triangle and estimate the magnitude of the joint contact force in terms of the shown magnitude of ground reaction force.
- (c) In the space provided below, show a free-body diagram of the leg-foot system that would be appropriate for a dynamics analysis of the system and write out the (2D) equation of rotational motion for this system in terms of all the appropriate parameters. (Define all dimensions in your free-body diagram). Assume accelerations are positive in the counter-clockwise direction.

3. (30 points total) Bone Cancer and Beam Theory

The Figure below shows a diaphysis cross-section affected by a metastatic tumor. When metastatic tumors develop in bones, they cause a defect — essentially a void — in the bone, reducing the ability of the bone to resist the forces and moments which act on it. We are typically interested in assessing the stresses in the bone with a defect to determine if they are dangerously high. If so, the surgeon will implant a stabilizing prosthesis so that the bone will not fracture unexpectedly in the future.

The cross-section shown below represents a diaphysis, assumed to be a hollow circular tube of cortical bone, with a defect in the middle of the cortex on the lateral aspect. For this model, we want to calculate the maximum tensile stress on the lateral aspect (at point a) of the cross-section for an applied compressive axial force, F , and a pure moment, M , which acts about an axis parallel to the X -axis causing tensile stresses on the lateral aspect. Assume that the axial force F acts at the center of the cross-section, denoted by point o . The inside and outside diameters of the cross-section are denoted by D_1 and D_2 , respectively, and the diameter of the defect is denoted by D . It is centered at point b in the middle of the cortex. Assume that the bone is symmetric about an axis passing through points a , b , and o . Specifically, do the following:

- (i) Determine the position of the neutral axis \bar{y} as a function of D , D_1 , and D_2 .
- (ii) Determine the moment of inertia I as a function of \bar{y} and any other parameters.
- (iii) Determine the tensile stress at point a as a function of F , \bar{y} , and any other parameters.



4. (20 points total) Miscellaneous

A. Sketch a stress-strain curve for cortical bone, comparing the compressive and tensile behaviors for longitudinal loading. Show a typical value of modulus, and the tensile and compressive strengths.

B. The NCAA Injury Surveillance System has reported that women college basketball players are three to four times more likely to sustain an ACL injury than their male counterparts. Can you suggest any biomechanical reasons for this? *Explain.*