ME/BIOE C176 Mid-Term Exam, Spring 2001 *Name:*______________________________

Wednesday, February 28, 9:30–11:00 AM, 2001.

Please write all answers in the space provided. If you need additional space, write on the back sides. Indicate your answer as clearly as possible for each question. *Write your name at the top of each page as indicated.*

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1. (15 points total) Forces and Moments at Joints
A. Estimate the force (in terms of body weight) acting in the erector spinae muscles (spine extensors) for holding a weight of one-tenth body-weight with arms outstretched and back bent. State all assumptions.

B. Development of abdominal pressure is thought to decrease loading of the spine, but this is controversial. Briefly explain this controversy using free-body diagrams to support your analysis.

2. (15 points total) Joint Dynamics

Figure 1 shows a simplified diagram of the head, rotating in the counter clockwise direction about the fixed point *o*, which is the center of the occipital condylar joint at the base of the head. The neck and torso are fixed, simulating a vehicle at full stop after hitting a rigid obstacle with the torso ideally restrained by a seat belt. In this situation, the unrestrained head will continue to move (*i.e.* rotate about its base) although the neck and torso are at rest.

Convert Figure 1 into a free-body diagram as follows. The (joint) contact force **J** acts at the joint center through point *o* and is purely vertical. The vertical (posterior) ligament force $\mathbf{F_v}$ acts at point *p*, a horizontal distance **d** posterior to the joint center; and the horizontal ligament force $\mathbf{F_h}$ acts through the joint center in the anterior-to-posterior direction. Besides the weight of the head **W**, assume that there are no other forces acting on this system. The center of mass of the head is at point *cg*, the velocity of this point is **u** in the anterior direction; the distance between points *o* and *cg* is **r**; points *o* and *cg* lie on the vertical Y-axis.

A. Show that the ratio of vertical to horizontal ligament forces is $\frac{F_v}{F_h} = \frac{I_o}{m\,d\,r}$.

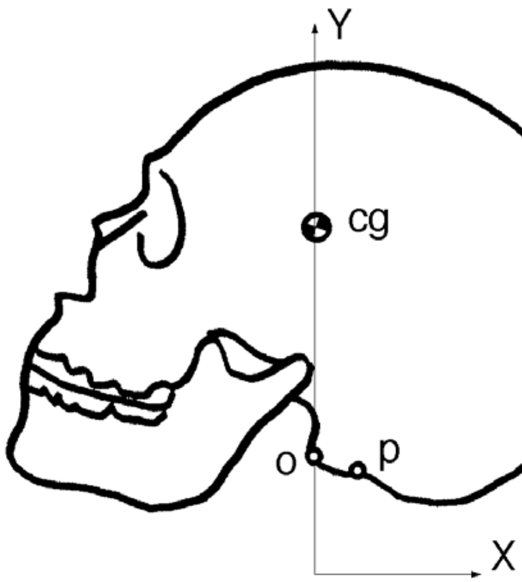

Figure 1: Schematic of the human skull, shown fixed at its base moving in pure rotation.

B. Why were no muscle forces included in this analysis?

3. (25 points) Joint Stability

A. One common mode of injury to the knee is rupture of the lateral collateral ligament as a result of an overload applied rapidly in the medial direction to the foot. Determine the magnitude of the medially-directed force *F* at the foot required to rupture the lateral collateral ligament assuming the strength of the ligament is 1500 N. Use the quadriceps and condylar forces and the dimensions as shown in Figure 2.

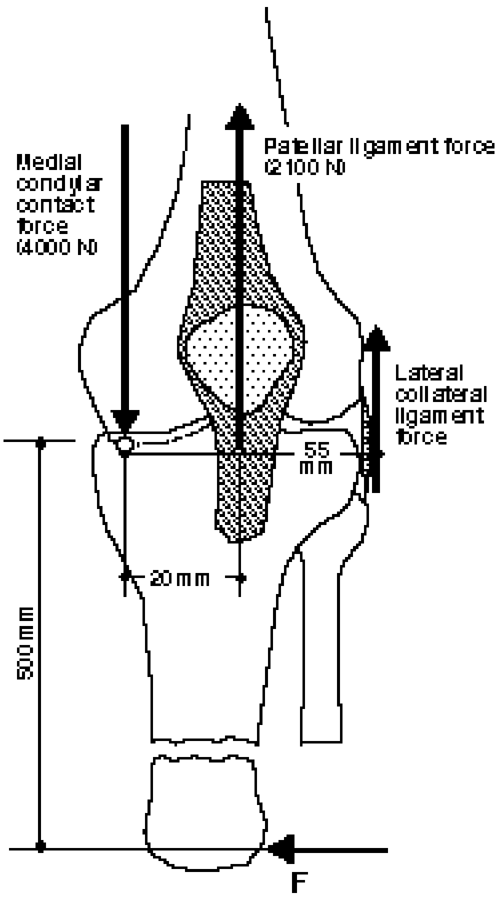

Figure 2

B.

If you had "knock-knees" (opposite of being bow-legged), would you expect bone stresses in your tibial plateau during gait to be higher on the medial or lateral side? Explain, using a free-body diagram.

4. (30 points total) Osteoporosis and Composite Beam Theory
With a fall to the side of the hip, a medially directed force F at the greater trochanter can generate a force J on the femoral head at an angle β to the femoral axis (as shown in Figure 3) and some distal equilibrating loads (not shown). The angle between the neck and femoral axis is θ. Of particular interest is the importance of the eccentricity of the trabecular bone with respect to the cortical bone within the femoral neck, *i.e.* the circular cross-section of trabecular bone is not centered within the circular cross-section of cortical bone in the femoral neck.

Write out the expression for the maximum tensile stress on the inferior aspect of section A–A for the loading conditions shown in Figure 3 in terms of the appropriate forces, angles, dimensions, and material properties, the position of the neutral axis $\bar{y}$, and the areal moments of inertia for the trabecular I_{trab} and cortical I_{cort} bone with respect to their centroids. Assume that composite beam theory applies, and neglect analysis of any shear stresses.

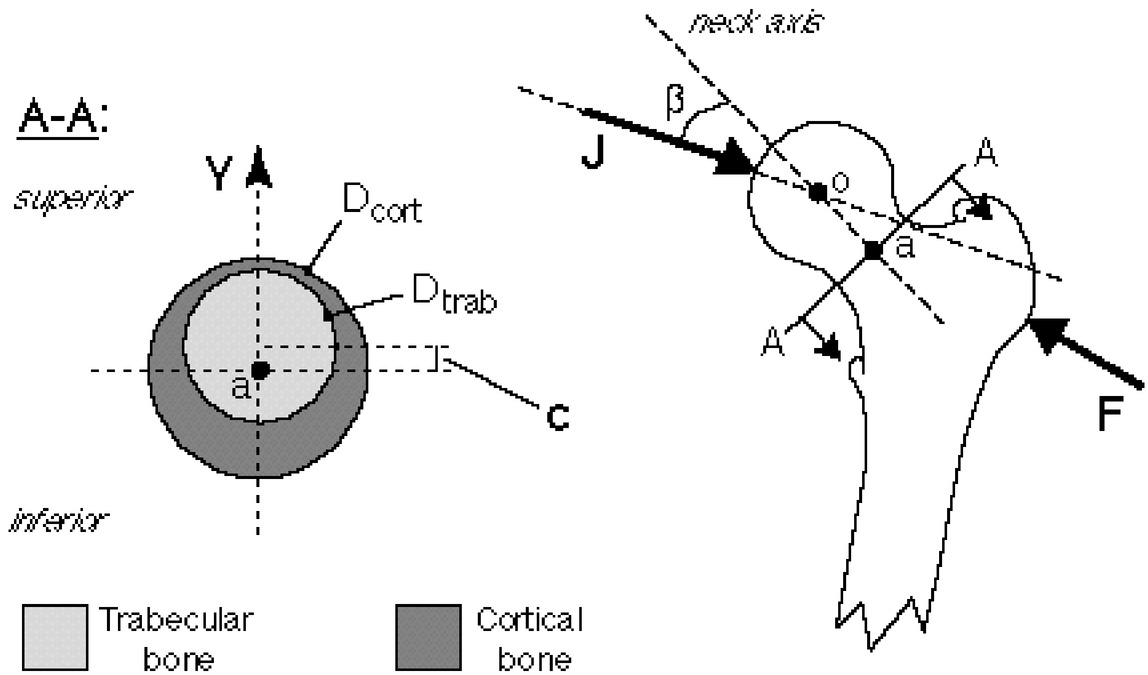

Figure 3: Left: Cross-section of the femoral neck at section A–A, showing the eccentricity of the trabecular bone with respect to the cortical bone and the correspondingly thicker cortical shell on the inferior aspect of the neck. a is the position of the center of the cortical bone area at the section. The distance between point a and the centroid of the trabecular bone is c, and the diameter of each type of bone is D_{cort} and D_{trab}, respectively, with corresponding moduli of E_{cort} and E_{trab}. **Right:** Model of the proximal femur when impacted at the greater trochanter. In response to the external trochanteric contact force F, an internal joint contact reaction force J is generated at the hip acting towards the center of the femoral head o as shown (making an angle β to the femoral axis). The equilibrium forces and moments that must act distally are not shown.

5. (15 points total) Fall Mechanics

A. Derive an equation for the velocity at impact at the hip during a sideways fall, assuming the falling body can be modeled as a single rigid slender uniform rod, rotating about the foot during the descent.

B. Based on this analysis and your answer to Question 4, what are the three body parameters that most increase risk of fracture during a fall?