Wednesday, October 9, 11:00 AM –12:30 PM, 2002.

Please write all answers in the space provided. If you need additional space, write on the back sides. Do not unstaple, remove, or add any pages. *Write your name at the top of each page as indicated.*

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1. (30 points total) Statics and Stability

(i) [5 points] Define each of the following:

joint contact force

joint reaction force

resultant force at the joint

(ii) [10 points] In a test for ligament stability, the patient sits down with the suspect leg not touching the ground (Figure 1). A horizontal force is applied to the proximal tibia as shown by the force at A, and then at the distal tibia at B. Both forces have the same magnitude, but are applied one at a time. Assume that the joint reaction force **J** (not shown) acts through point *a* within the knee joint, as shown, for both loading configurations, and that the direction of the quadriceps muscle force **Q** is the same for each loading case. <u>What is the approximate direction of the joint reaction force for each load case (free body diagrams will suffice as an answer)?</u> Treat this as a 2D static problem and ignore the weight of the leg.

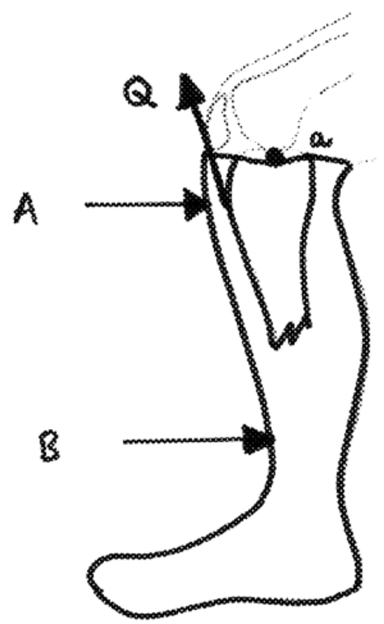

Figure 1: Free-body diagram (incomplete) of the lower leg for two loading scenarios. The joint reaction force **J**, acting at point *a*, is not shown.

(iii) [10 points] Referring to part (ii), what are the stability mechanisms for each load case? Assume that the geometry of the articulating surface of the proximal tibia is as shown in Figure 1.

(iv) [5 points] How might this test be used clinically to examine the behavior of the cruciate ligaments?

2. (25 points total) Dynamic Analysis

A dumbbell is held in the hand during a curl exercise, and quickly raised toward the shoulder in a circular motion (angular acceleration α and angular velocity ω) about the elbow joint. The wrist is fully rigid during this motion and the elbow does not move. At some instant before the forearm reaches the horizontal position, it is at an angle θ to the vertical and is still accelerating. Treat this as a 2D planar rigid body problem.

(i) [10 points] Draw a fully labeled free body diagram of the forearm/hand/dumbbell system. Include all accelerations, shown in their assumed positive directions. Use resultant loads at the elbow joint.

(ii) [15 points] Write out the three equations of motion for this problem. Explain any nomenclature not evident from your free-body diagram — and pay attention to subscripts.

3. (35 points total) Composite Beam Theory

(i) [20 points] Derive from first principles the following formula for the location of the neutral axis in a composite beam:

$$\hat{y} = \frac{\Sigma\ E_i A_i \overline{y_i}}{\Sigma\ E_i A_i}$$

<u>State clearly and explicitly all your assumptions.</u>

(ii) [15 points] For the cross-section shown below, write out an expression for the stress at point *A* in terms of the dimensions *h* and *b*, the applied pure bending moment *M*, and the modulus *E*. Assume point A is bending in tension. Note that the top material has a central rectangular hole in it, of width *b/2* and height *h* such that the neutral axis is located at a distance *83h/58* above point *A*.

Hint: There is no need to work through all the algebra — just write out the appropriate expression in terms of the above variables.

3. (10 points total) Bone Mechanics and Osteoporosis

(i) **[3 points]** Sketch a graph of ultimate tensile strength vs. porosity for cortical bone. Give some typical values on each axis.

(ii) **[3 points]** Plot a single graph of yield strength vs. density for trabecular bone, showing behavior for both compressive and tensile loading (just show trends — no need to add values on the axes).

(iii) **[4 points]** How does the World Health Organization define osteoporosis, and what is one limitation of such a definition?