Mathematics 1B. Fall Semester 2006 **Professor: Daniel Tataru**

Midterm 2

(20) 1. Determine the interval of convergence of the following series. Do they converge at endpoints ?

$$a) \qquad \sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{\sqrt{n}\ 4^n}$$

(20) 2. Find the Maclaurin series expansion of the following functions. Determine where the expansions are valid (i.e. for what values of x they converge).

$$a) \qquad f(x) = \frac{x}{x^2 + x - 2}$$

$$b) \qquad f(x) = \sqrt{1 + x^2}$$

(20) 3. a) Find the third order Taylor polynomial of $\tan x$ at $\pi/4$.

b) Find the Maclaurin series for a function f which solves the differential equation

$$f''(x) = xf(x), \qquad f(0) = 1,\ f'(0) = 0$$

What is the radius of convergence ?

(20) 4. Sketch the direction field of

$$y' = y^3 - y$$

and determine the equilibrium solutions. Are they stable ?

(20) 5. Solve the initial value problems

$$a) \qquad \frac{dx}{dt} = 2t(1+x^2), \qquad x(0) = 0$$

$$b) \qquad \frac{dx}{dt} = x + \sin t, \qquad x(0) = 1$$