Midterm 1

(20)   1. Evaluate the following (indefinite) integrals

$a) \int e^{\sqrt{x}} dx$

$b) \qquad \int x \tan^2 x \, dx$

(20)   2. Evaluate the following (definite) integrals:

a) $\int_{-\infty}^{\infty} \dfrac{4x^2}{x^4+4}\,dx$

b) $\int_{0}^{\pi/2} \dfrac{\cos x}{\sqrt{1+\sin^2 x}}\,dx$

(20)   3. a) Suppose that $f(x)$ is a function defined on $[a, b]$. State the formula for the area of the surface of revolution obtained by rotating the graph of f around the y axis.

b) Find that area in the case when $f(x) = 3x^{1/3}$ and $a = 0$, $b = 1$.

(20)   4. Determine (providing an explanation) the convergence or divergence of the following series:

$$a) \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

$$b) \sum_{n=1}^{\infty} \frac{1 + (-1)^n n}{n^2 + 2n}$$

$$c) \sum_{n=1}^{\infty} \frac{(n!)^2}{e^{n^2}}$$

(20)   5. a) Estimate the error in approximating the following series by the sum of its first 10 terms:

$$\sum_{n=1}^{\infty} \frac{1}{n^4 + n^2}$$

b) Estimate the partial sums of the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

c) Compute the sum of the series

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$