University of California, Berkeley

Math 1B Midterm 2 Solutions

Slobodan Simić, Spring 2012

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Name and SID:

GSI (circle one): Aditya Adiredja, Shuchao Bi, Boaz Haberman,

Weihua Liu, Peter Mannisto, Rene Quilodran, Zvi Rosen,

Eugenia Rosu, Per Stinchcombe, Zack Sylvan, Michael Wan

	Score
1	
2	
3	
4	
5	
Total	

Instructions. Read the problems very carefully to be sure you understand the statements. Justify your answers. Show all your work as clearly as possible and $\boxed{circle}$ the final answer to each problem. When giving explanations, write complete sentences. If you have any questions, please ask any of the proctors. When you are done with the exam, please hand it to the nearest (not necessarily your own) GSI. If you finish early, please leave quietly.

1. (20 points) For each of the following statements, determine if the conclusion ALWAYS follows from the assumptions, if the conclusion is SOMETIMES true given the assumptions, or if the conclusion is NEVER true given the assumptions. You do not need to show any work or justify your answers to these questions – only your circled answer will be graded.

(a) If $a_n \to 0$, as $n \to \infty$, then $\quad (-1)^n a_n$ is convergent.

(b) If $\sum a_n$ is convergent and $a_n > 0$ for all n, then $\sum \sqrt{a_n}$ is convergent.

(c) If $\quad a_n$ converges, then $(a_1 + a_2 + \cdots + a_n)/n \to 1$, as $n \to \infty$.

(d) If (b_n) is convergent sequence, then the series $\quad (b_n - b_{n+1})$ is convergent.

(e) If $\sum(-2)^n c_n$ is divergent, then $\sum 3^n c_n$ is convergent.

2. (20 points) Compute the limits of the following sequences:

(a) $a_n = \sqrt{\dfrac{12n^4 + \pi n}{3n^4 - n^2 + 2012}}$

(b) $b_n = \sqrt{n} \arctan \dfrac{\pi}{\sqrt{n}}$

3. (20 points) For which values of p, where $-\infty < p < \infty$, is the series

$$\sum_{n=1}^{\infty} \frac{\sin\left(\dfrac{1}{n^2}\right)}{n^p}$$

convergent?

4. (20 points) For which values of x does the series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{(n+1)\ 3^n}$$

converge?

5. (20 points) Consider the function f defined by

$$f(x) = \begin{cases} \dfrac{e^{x^2} - 1 - x^2}{x^3} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

(a) Find the MacLaurin series for f. What is its radius of convergence?

(b) Express

$$\int_0^1 f(x)\,dx.$$

as the sum of an infinite series.