UCB Math 1B, Fall 2009: Final Exam

Prof. Persson, December 18, 2009

Name: _____

SID: _____

Section: Circle your discussion section below:

Sec	Time	Room	GSI
01	MW 8am - 9am	75 Evans	G. Melvin
02	MW 8am - 9am	5 Evans	T. Wilson
03	MW 10am - 11am	75 Evans	D. Cristofaro-Gardiner
04	MW 10am - 11am	3113 Etcheverry	E. Kim
05	MW 11am - 12pm	81 Evans	G. Melvin
06	MW 12pm - 1pm	5 Evans	T. Wilson
07	MW 1pm - 2pm	2 Evans	A. Tilley
09	MW 2pm - 3pm	247 Dwinelle	D. Cristofaro-Gardiner
10	MW 3pm - 4pm	4 Evans	E. Kim
11	MW 4pm - 5pm	3113 Etcheverry	A. Tilley
12	TT 11:30am - 2pm	230C Stephens	L. Martirosyan

Other/none, explain: _____

Grading

1	/ 10
2	/ 10
3	/ 10
4	/ 10
5	/ 15
6	/ 15
7	/ 15
8	/ 15
	/100

Instructions:

- One double-sided sheet of notes, no books, no calculators.
- Exam time 180 minutes, do all of the problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.

1. (10 points) Solve the initial value problem $y' + y\cos x = y^2 \cos x,\ y(0) = 2.$

2. (10 points) Find a particular solution of the differential equation

$$y'' + y = \csc x, \quad 0 < x < \pi/2.$$

3. (10 points) Find the interval of convergence, including determination of the convergence at the end points, for the power series

$$\sum_{n=2}^{\infty} \frac{(-2)^n (x+1)^n}{n(\ln n)^2}.$$

4. (10 points) Consider the function $f(x) = \int_0^x \cos(t^2)\, dt$.

(a) Find the Maclaurin series of $f(x)$.

(b) Estimate the accuracy of the approximation $f(x) \approx T_5(x)$ (the Maclaurin polynomial of degree 5) for $|x| \leq 1/2$.

5. (15 points) Evaluate the integral or show that it is divergent (continued on next page).

(a) $\displaystyle\int_1^{\infty} \frac{\ln x}{x^2}\, dx$

(b) $\displaystyle\int_0^{1} \frac{e^{1/x}}{x^3}\, dx$

(continued from previous page)

(c) $\displaystyle\int_0^{\pi/2} \frac{\cos t}{(1+\sin^2 t)^{5/2}}\, dt$

6. (15 points) Determine if the series below are absolutely convergent (AC), conditionally convergent (CC), or divergent (D) (continued on next page).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^3+3n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} n \sin(n^{-3/2})$

(continued from previous page)

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\sqrt{n+1}}-\frac{1}{\sqrt{n}}\right)$

7. (15 points) Consider an undamped mass-spring system with mass m and spring constant k, subject to an external force $F(t) = F_0 \cos \omega t$ where $\omega = \sqrt{k/m}$. Find the position $x(t)$ of the mass at time t, relative to the equilibrium position, given that it starts from the position $x(0) = x_0$ with velocity $x'(0) = v_0$.

8. (15 points)

(a) Use power series methods to solve the initial value problem $y'' - xy' - 2y = -4x^2$, $y(0) = 1$, $y'(0) = 1$.

(b) Write the solution in terms of elementary functions.