MATH 1B

Lec. 3, Spring 2010
Midterm 2

1

Let

$$a_n = (-1)^n \frac{2n^2 + 3}{n^2 + n + 1}.$$

a) Determine whether the sequence $\{a_n\}_{n=1}^{\infty}$ converges or diverges. If it converges, find what it converges to. If it does not converge, state the reason why.

b) Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges. If it converges, find what it converges to. If it does not converge, state the reason why.

2

Find the sum $\sum_{n=1}^{\infty} a_n$, where

$$a_n = \frac{1}{n^3} - \frac{1}{(n+1)^3}.$$

by computing the partial sums and taking a limit

3

Use the integral test to determine whether

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n},$$

is convergent, or not. (Note that the series meets the conditions for the integral test.)

4

Determine if the series

$$\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{4^n}.$$

is absolutely convergent, conditionally convergent or divergent.

5

Determine if the series

$$\sum_{n=2}^{\infty}(\frac{n}{\ln n})^n$$

is convergent or divergent.

6

Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=0}^{\infty} nx^n.$$

Which function of x does this sum up to? (Recall that the series is related in a simple way to another series you know well).

7

Find Taylor/MacLaurin series expansion for

$$f(x) = \exp(2x),$$

around $x = 0$, using the definition of the Taylor/MacLaurin series. Can you check your answer against something you already know?