MATH1B MIDTERM 2 (ZWORSKI)

Solve each problem on a separate sheet writing your name and section number on each sheet. Do not panic! Good luck!

Problem 1. (16 points)
a) Determine the radii of convergence of the following two power series:

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{(n+1)^{\frac{3}{2}}}, \quad \sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{(n!)^2} x^n .$$

b) Do the series in part b) converge at the *end points*? Provide clear arguments for your answers.

Problem 2. (6 points) Suppose that the power series

$$\sum_{n=0}^{\infty} a_n (x \quad 2)^n ,$$

converges at $x = 6$. What can you say about its convergence at $x = -1$?

Problem 3. (8 points) Evaluate the following integral as an infinite series:

$$\int_0^1 \left(1 - \left(\frac{x}{3}\right)^4\right)^{\frac{1}{2}} dx .$$

Remark/Hint: If you use generalized binomial coefficients, provide their definition.

Problem 4. (6 points)
a) Write out the n-th Taylor polynomial, $T_n(x)$, of $f(x)$ at $x = 0$.
b) Give an estimate for $R_n(x) = f(x) - T_n(x)$, for $|x| \leq 1$.
c) Prove the estimate for $R_0(x)$ $(f(x) = T_0(x) + R_0(x))$.

Problem 5. (12 points)
a) Find the 4th degree Taylor polynomial (centered at 0) of the function

$$x^2 \cos x ,$$

b) Give a numerical estimate for the remainder valid in $|x| \leq 1$.

Problem 6. (12 points)
a) Find the general solution of the following differential equation:

$$y' + x^2 y = x^2 .$$

b) Find the solution solution of *the same equation* with the boundary condition $y(0) = 0$.

c) Match the following direction fields with differential equations (**Please do not guess – points will be taken off**)

1) $y' = y^2(1 - x^3)$
2) $y' = \cos x \cos y$
3) $y' = y^2 - x^2$
4) $y' = (1 - y^2)\cos x$