Mathematics 1B, Fall 2001 — M. Christ
Midterm Examination #2

Instructions.
1. Closed book exam. No formula sheets or notes are permitted except those on the following page. Calculating devices are not allowed.
2. Do all work on the sheets provided. There is one blank sheet.
3. **Show work and/or reasoning to receive credit.**
4. Important:
Place your final answers (but not your reasoning) **in the boxes, where provided.**

NAME:

Your TA' ... USSION SECTION time:

Problem	Point Value	
1	$18 = 3 \times 6$	
2	7	
3	$12 = 6 + 6$	
4	7	
5	6	
Total	50	

Formulas

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad \cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad |R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n. \qquad \ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$\lim_{n \to \infty} \frac{x^n}{n!} = 0 \text{ for every } x.$$

Please note that no formulas related to the various convergence tests for numerical series are provided; you are expected to know these. You are expected to know what the various symbols mean, in particular, what the notation $\binom{k}{n}$ means, and what "M" in the bound for R_n signifies. (It is okay to use the notation $\binom{k}{n}$ in your solutions if it comes up.)

(1) Determine whether each of the following series converges, or diverges. Justify your reasoning. State which tests you use. (**Do not** list tests which are inconclusive, only those which contribute to your conclusion.)

(1a) $$\sum_{n=3}^{\infty} \frac{1}{2n^3 - n^2 - 5n}$$

(1b) $$\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n^2)}{n^2}$$

(1c) $\sum_{n=1}^{\infty} \frac{n!}{\sqrt{n^n}}$

(2) Consider the infinite series $\sum_{n=1}^{\infty} n^{-2}$. Find the *smallest* N such that the partial sum s_N approximates the sum of the series within 10^{-3}. (You will receive partial credit for finding *some* N that works, even if it is not the smallest.)

(3a) Find the Maclaurin series for $\arcsin(x)$. You may use the fact that the derivative of $\arcsin(x)$ is $(1-x^2)^{-1/2}$.

(3b) Suppose we know that an infinite series $\sum_{n=1}^{\infty} c_n(-2)^n$ converges. For which values of x can we conclude that $\sum_{n=1}^{\infty} c_n(x-1)^n$ must converge? For which x can we conclude that the latter series diverges?

(4) For which x does the Taylor series for the function $f(x) = \sin(x)$, at $\pi/2$, converge to $\sin(x)$? (Stewart uses the expressions "centered at", and "about" as synonyms for "at" in this context.) Justify your answer in detail.

(5) Let $a_1 = \sqrt{2}$, and $a_{n+1} = \sqrt{2a_n}$ for all $n \geq 1$. Show that $\{a_n\}$ converges, and find its limit. Justify your reasoning; you may use any theorems from the text.