Midterm 2

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	Score
1	
2	
3	
4	
5	
Total	

EXPLAIN YOUR WORK

1. (20 points) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function and define

$$h(x) = f(x^2) - f\left(\frac{1}{x^2}\right) + f(e^{2(x-1)}),$$

for $x = 0$. If $f'(1) = 1$, compute $h'(1)$.

2. (20 points) A curve C is defined by the equation

$$x^4 + y^4 = \cos^4 y + xy.$$

Find the equation of the tangent line to C at the point of intersection of C with the positive x-axis.

3. (20 points) (a) Show that the equation $x^3 + 3x + 2 = 0$ has a unique root and that it lies in the interval $(-1, 0)$.

(b) Find the absolute extrema of the function

$$f(x) = \frac{x^3 - 1}{x^2 + 1}$$

on the interval $[\ 1, 2]$.

4. (20 points) (a) If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function and $f'(x) = c$, for all $x \in \mathbb{R}$, where c is a constant, what can be said about f?

(b) Assume $f''(x) = 0$, for all $x \in \mathbb{R}$. If $f(0) = -1$ and $f'(0) = 1$, compute f.

5. **(20 points)** Let
$$f(x) = e^{-x^2 + 2x}.$$

(a) Find the intervals of monotonicity and extrema of f.

(b) Find the intervals of concavity and inflection points of f.

(c) Find the horizontal asymptotes of f.

(d) Sketch the graph of f.