University of California, Berkeley

**Math 1A, Section 3 (Prof. Simić), Fall 2011**

# Midterm 1

September 22, 2011

## Name and SID:

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| | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

Explain your work

**1. (20 points)** Let us write $\exp(x)$ for $e^x$. Define

$$f(x) = \exp(1 + \exp(2 + 3x)).$$

(a) Find the domain and range of $f$.

(b) Show that $f$ is 1–1.

(c) Compute $f^{-1}$.

**2. (20 points)** Sketch the graph of the following functions. Start with the graph of an "easy" function and apply the appropriate transformations.

(a) $f(x) = |x^2 + 2x|$.

(b) $g(x) = \frac{x}{x-2}$.

**3. (20 points)** For each of the following limits, compute it if it exists or explain why it doesn't exist. Use only limit laws and theorems from the book but not informal arguments (such as "this is big and that is big, so their product is big"). Explicitly mention those theorems and limit laws by name (not by their number in the book). In (d), $[x]$ denotes the largest integer $\leq x$.

$$\text{(a)} \quad \lim_{x\to\infty} \frac{1+2x+3x^2+4x^3+5x^6}{x^2+x^6}.$$

$$\text{(b)} \quad \lim_{x\to 0} \frac{e^{-1/x^2}}{1+e^{-1/x^2}}$$

$$\text{(c)} \quad \lim_{x\to 2} \frac{x^4+3x^3-10x^2}{x-2}$$

$$\text{(d)} \quad \lim_{x\to 1} x[x].$$

**4. (20 points)** (a) Using limit laws and theorems from the book, compute

$$\lim_{x \to 0} x^2 \cos \frac{1}{x}.$$

(b) Using the precise ($\varepsilon$-$\delta$) definition of the limit, prove that your answer in (a) is correct. (**Hint:** cosine is bounded.)

**5. (20 points)** Let

$$f(x) = \begin{cases} \arctan|x| & \text{if} \quad x < 0, \\ \arcsin(x-1) & \text{if} \quad 0 \le x < 2, \\ \frac{\pi}{4}x & \text{if} \quad x \ge 2. \end{cases}$$

(a) Find all the points where $f$ is discontinuous.

(b) Explain why $f$ is continuous at all other points.

(c) Approximately sketch the graph of $f$.