Mathematics 1A, Section 2 December 13, 2001
Sarason

Final Examination

Name (printed): _____

Signature: _____

SID: _____

GSI: _____

Section Time: _____

Put your name on every page.

Closed book except for crib sheet. No calculators.

SHOW YOUR WORK. Cross out anything you have written that you do not want the grader to consider.

The points for each problem are in parentheses. Perfect score = 200.

0	
1	
2	
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10	
Total	
Grade Points	

0. (10) Suppose $f(x) = 1 + x^2$. If $g = f \circ f$ and $h = f \circ g$, what are $g'(1)$ and $h'(1)$?

1. (15) Find f' for the given functions f.

(a) $f(x) = \sqrt{1 + \sqrt{1 + x^2}}$ (b) $f(x) = e^{\sqrt{\ln x}}$ (c) $f(x) = \int_0^x e^{-(x-t)^2} dt$

Name _____

2. (15) Evaluate the limits.

(a) $\lim_{x \to 0} \dfrac{\cos 5x - 1}{x^2}$ (b) $\lim_{x \to \infty} x(\sqrt{x^2 + 1} - x)$ (c) $\lim_{x \to 0} \dfrac{x^x - 1}{\sqrt{x}}$

Name _____

3. (15) What are the maximum and minimum values of the function $f(x) = (2x+1)e^{-x^2}$ on the interval $[0, 1]$?

Name _____

4. (15) Perform the integrations.

(a) $\int e^x \cos(e^x) dx$ (b) $\int \frac{x^2+x+1}{x-1} dx$ (c) $\int_0^1 (1-x^{1/3})^{3/2} dx$

Name _____

5. (15) Find the area of the region bounded by the parabola $y = x^2$, the y-axis, and the tangent line to the parabola at the point $(2, 4)$.

6. (15) What is the volume of the solid one obtains by revolving about the y-axis the region bounded by the right branch of the hyperbola $x^2 - y^2 = 1$ and the line $x = 2$?

Name _____

7. (20) What is the volume of the solid one obtains by revolving about the x-axis the region below the line $y = 4$ and above the curve $y = e^x + e^{-x}$?

Name _____

8. (30) For the function $f(x) = \dfrac{x}{(1-x)^2}$:

(a) Determine the intervals of increase, the intervals of decrease, and the local maxima and minima.

(b) Determine the intervals of upward and downward concavity, and the points of inflection.

(c) Sketch the graph.

Name _____

9. (30) Ship A is headed east at 30 miles per hour. Ship B is headed north at 30 miles per hour. At noon ship B is 30 miles due south of ship A.

   (a) What is the relative position of the ships when the distance between them is smallest?

   (b) At what rate is the distance between the ships increasing at 3 p.m.?

Name _____

10. (20) A hemispherical bowl had radius 6 inches at its top. Water is dripping into the bowl at a rate of 2 cubic inches per minute. At what rate is the water level rising when the water is 4 inches deep?