Mathematics 1A, Fall Semester 2003
Instructor: Garth Dales
September 25, 2003

Midterm Examination 1

Your Name: _____

Your SID: _____

Directions:

- Do not open your exam until your are instructed to do so.
- You may not use any external aids during the exam: NO books, NO lecture notes, NO formula sheets, NO cell phones.
- Answers without explanation will not receive credit. *You must show and justify your work.* If necessary, use the backs of the pages or the extra pages attached to your exam, and indicate you have done so.
- When time is called, you must stop working and close your exam.
- All questions are worth 12 points each (for a total of 60 points).

Good Luck!

Score

1. (a) *(4 points)* Let f be a function defined near $x = a$. Write down the definition (involving ϵ and δ) of the following:

$$\lim_{x \to a} f(x) = \ell$$

(b) *(4 points)* Prove *from the ϵ-δ definition of limit* that

$$\lim_{x \to 2} x^2 = 4$$

(c) *(4 points)* Evaluate the following limits by using the rules of limits (state at each step which rule you are using):

i.

$$\lim_{x \to 0} x^2 \left[\sin\left(1/x\right) + \sin\left(e^x\right)\right]$$

ii.

$$\lim_{x \to \infty} \frac{3x^2 + 2x - 7}{2x^2 - x + 17}$$

2. (a) *(3 points)* Let $f : S \to T$ be a function. Explain what it means for f to be one-to-one.

(b) *(4 points)* Draw the graphs of the following functions on one diagram:

$$y = \exp(x), \quad y = x, \quad y = \log(x)$$

[Show where the curves cross each of the two axes, and the relationship between the curves.]

(c) *(5 points)* Let

$$f(x) = \frac{2 + x - x^2}{12 - 7x + x^2}$$

(1) What is the (maximal possible) domain of f? (2) Calculate

$$\lim_{x \to 3^+} f(x), \quad \lim_{x \to 3^-} f(x), \quad \lim_{x \to \infty} f(x), \quad \lim_{x \to -\infty} f(x)$$

(3) Write down the equations of one vertical asymptote and one horizontal asymptote to the curve $y = f(x)$.

3. (a) *(6 points)* Find the number c such that the following function f is continuous at $x = 3$:

$$f(x) = \begin{cases} \frac{c|x-3|}{x-3} & \text{if } x < 3 \\ x^2 - cx - 5 & \text{if } x \geq 3 \end{cases}$$

(b) *(6 points)* Let f be a function which is continuous at $x = a$, and let g be a function which is continuous at $y = f(a)$. Prove from the precise definition that the composition $g \circ f$ is continuous at $x = a$.

4. (a) *(6 points)* State without proof the Intermediate Value Theorem for a function f defined on a closed interval $[a, b]$.

(b) *(6 points)* Use the Intermediate Value Theorem to show that there is a number x in the interval $[0, 1]$ such that

$$e^x = 2 - x$$

5. (a) *(4 points)* Let f be a function which is differentiable at $x = a$. Write down:

i. the definition of $f'(a)$ in terms of a limit;

ii. the formula for the tangent to the curve $y = f(x)$ at the point $(a, f(a))$.

(b) *(8 points)* Calculate the derived function f' of each of the following functions f. (In each case, you should use standard rules of differentiation.)

i. $f(x) = e^{3x} \sin(2x)$

ii. $f(x) = \sin^3(x^2 + 1)$

iii. $f(x) = \log(3x^2 + 1)$

iv. $f(x) = \tan^{-1} x$