Second Midterm Examination
Friday April 3 2009
Closed Books and Closed Notes

Question 1 *A Linkage System*

As shown in Figure 1, a mechanical linkage consists of a system of 4 particles which are connected by a set of identical massless rods each of length L to a central point C. The masses of the particles are identical $m_1 = m_2 = m_3 = m_4$. The particle of mass m_3 is connected to a fixed point O by a linear spring of stiffness K and unstretched length L_0. The mechanism moves on a smooth horizontal plane. A gravitational force acts normal to this plane.

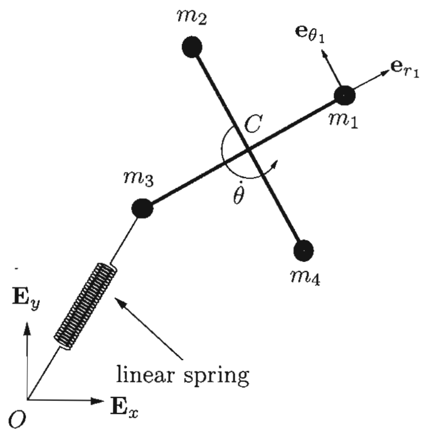

Figure 1: A system of particles moving on a smooth horizontal plane.

(a) *(6 Points)* Starting from the representations for the position vector of the center of mass C and the position vector of the particle of mass m_1,

$$\mathbf{r} = x\mathbf{E}_x + y\mathbf{E}_y, \qquad \mathbf{r}_1 = L\mathbf{e}_{r_1} + \mathbf{r}, \tag{1}$$

establish expressions for the linear momentum $\mathbf{G}_1$ and angular momenta relative to O and C of the particle of mass m_1.

(b) *(4 Points)* For the system of 4 particles,

$$\mathbf{G} = m\left(\dot{x}\mathbf{E}_x + \dot{y}\mathbf{E}_y\right), \qquad \mathbf{H}_C = mL^2\dot{\theta}\mathbf{E}_z, \tag{2}$$

where $m = m_1 + m_2 + m_3 + m_4$ and $\dot{\theta} = \dot{\theta}_i$ $(i = 1, 2, 3, 4)$. Using (2), what is $\mathbf{H}_O$?

(c) *(3 Points)* Draw a free-body diagram of the system of particles. In your solution, give a clear expression for the spring force.

(d) *(2 Points)* Using a balance of linear momentum for each particle show that the normal force acting on each particle is equal and opposite to the gravitational force.

(e) *(5 Points)* Show that $\mathbf{H}_O$ is conserved. In addition, show that $\mathbf{H}_C$ is not conserved.

(f) *(5 Points)* Starting from the work-energy theorem for a system of particles, show that the total energy E of the system of particles is conserved.

Question 2

A System of Two Particles

25 Points

As shown in Figure 2, a particle of mass m_1 is at rest and is attached to a smooth horizontal surface by two identical linear springs of stiffnesses K and unstretched lengths L_0. At time t = 0, a particle of mass m_2 traveling with a velocity vector $v_0\mathbf{E}_x$ impacts the particle of mass m_1. After the collision both particles adhere to each other, and can be considered as a particle of mass $m_1 + m_2$.

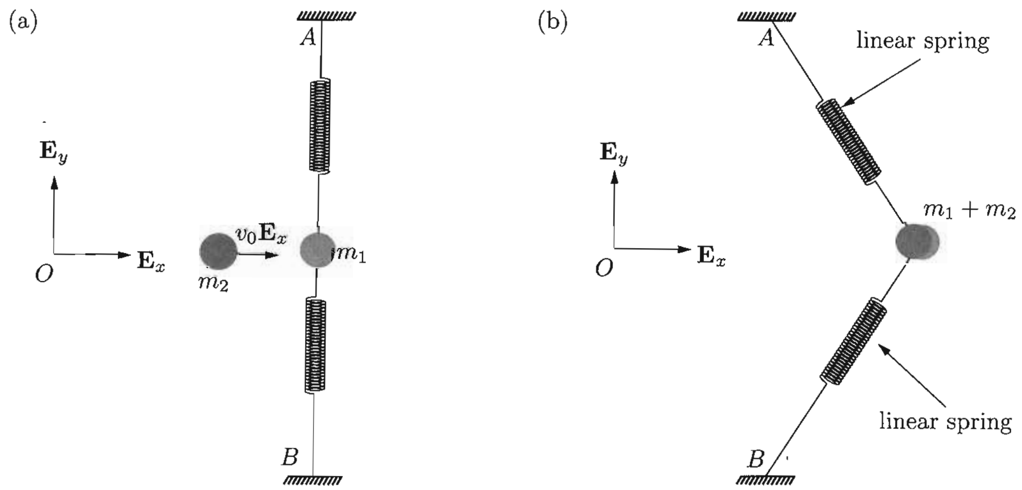

Figure 2: A system of two particles: (a) Prior to impact at $t = 0$, and (b) following the impact. The position vectors of the fixed points A and B are $\mathbf{r}_A = H\mathbf{E}_y + W\mathbf{E}_x$ and $\mathbf{r}_B = -H\mathbf{E}_y + W\mathbf{E}_x$, respectively

(a) *(4 Points)* Starting from the representation

$$\mathbf{r}_1 = (x + W)\,\mathbf{E}_x, \tag{3}$$

where W is a constant, establish representations for the linear momentum, kinetic energy, and acceleration of the particle of mass $m_1 + m_2$ after the collision.

(b) *(5 Points)* Show that the velocity of the particle of mass $m_1 + m_2$ immediately following the collision is

$$\dot{x}(t = 0) = \frac{m_2}{m_1 + m_2} v_0. \tag{4}$$

Verify that the kinetic energy of the system is not conserved during the collision.

(c) *(6 Points)* Draw a freebody diagram of the particle of mass $m_1 + m_2$ following the collision. Give clear expressions for the spring forces acting on the particle.

(d) *(5 Points)* Consider the system after impact. Starting from $\dot{T} = \mathbf{F} \cdot \mathbf{v}$ for a single particle, show that the total energy E of the particle of mass $m_1 + m_2$ is conserved. In your solution, give an expression for E.

(e) *(5 Points)* Following the impact of the particle of mass m_2, if $H = L_0$, show that the maximum displacement x_{max} of the particle of mass $m_1 + m_2$ is given by

$$x_{\text{max}}^2 = \left(\left(\sqrt{\frac{m_2}{2K} \left(\frac{m_2}{m_1 + m_2} \right)} \right) v_0 + H \right)^2 - H^2. \tag{5}$$