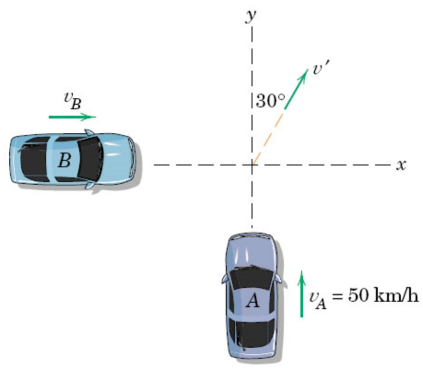
Department of Mechanical Engineering
University of California at Berkeley
ME 104 Engineering Mechanics II
Spring Semester 2007

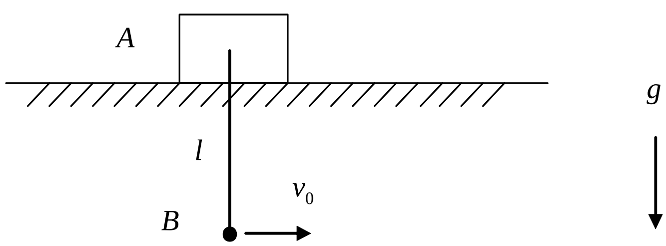
Instructor: F. Ma
Midterm Examination No. 2 April 6, 2007

The examination has a duration of 50 minutes.
Answer ALL questions.
All questions carry the same weight.

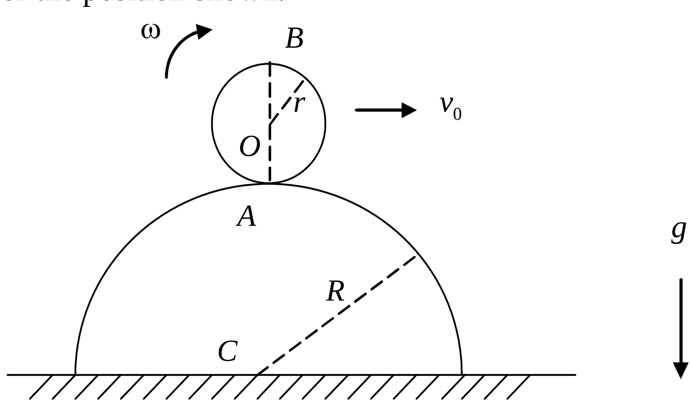
1. Two cars collide at right angles in the intersection of two icy roads. Car *A* has a mass of 1200 kg and car *B* has a mass of 1600 kg. The cars become entangled and move off together with a common velocity v' in the direction indicated. If car *A* was traveling 50 km/h at the instant of impact, compute the corresponding velocity of car *B* just before impact.



2. Ball *B*, of mass m_B, is suspended from a cord of length l attached to cart *A*, of mass m_A, which can roll freely on a frictionless horizontal track. If the ball is given an initial horizontal velocity v_0 while the cart is at rest, determine (a) the velocity of *B* as it reaches its maximum elevation, (b) the maximum vertical distance h through which *B* will rise. It is assumed that $v_0^2 < 2gl$.



3. The smaller cylinder rolls on the stationary larger cylinder without slipping. The speed of the center of the rolling cylinder is constant. Determine the acceleration of the point of contact *A* and the point *B* for the position shown.



1. This is a plastic impact with $e = 0$. Linear momentum is conserved.

$$\Delta G_x = 0$$

$$\Rightarrow \quad m_B v_B + 0 = (m_A + m_B) v' \sin 30°$$

$$\Rightarrow \quad 1600 v_B = 2800 v'(0.5) \qquad (1)$$

$$\Delta G_y = 0$$

$$\Rightarrow \quad m_A v_A + 0 = (m_A + m_B) v' \cos 30°$$

$$\Rightarrow \quad 1200(50) = 2800 v'(0.866)$$

$$\Rightarrow \quad v' = 24.7 \qquad (2)$$

From equation (1),

$$v_B = 21.7 \text{ km/h}$$

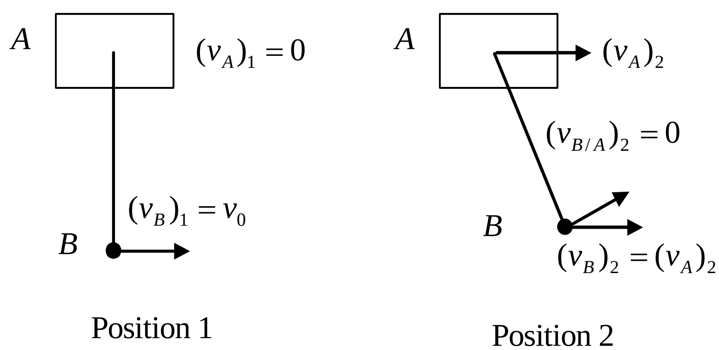
2. (a) When ball *B* reaches its maximum elevation in position 2, its velocity $(\mathbf{v}_{B/A})_2$ relative to cart *A* is zero. Since *A* is translating horizontally,

$$(\mathbf{v}_B)_2 = (\mathbf{v}_A)_2 + (\mathbf{v}_{B/A})_2 = (\mathbf{v}_A)_2$$

Since system linear momentum is conserved,

$$\Delta G_x = 0 \quad \Rightarrow \quad m_B v_0 = (m_A + m_B)(v_B)_2$$

$$\Rightarrow \quad (v_B)_2 = \frac{m_B}{m_A + m_B} v_0$$



(b) The energy of the system is conserved.

$$\Delta T + \Delta V_g = 0$$

$$\Rightarrow \quad \frac{1}{2} m_A (v_A)_2^2 + \frac{1}{2} m_B (v_B)_2^2 - \frac{1}{2} m_B v_0^2 + m_A g l + m_B g h - m_A g l = 0$$

$$\Rightarrow \quad h = \frac{v_0^2}{2g} - \frac{m_A + m_B}{m_B} \frac{(v_B)_2^2}{2g}$$

Using the expression of $(v_B)_2$ found earlier,

$$h = \frac{m_A}{m_A + m_B} \frac{v_0^2}{2g}$$

The assumption $v_0^2 < 2gl$ ensures that

$$h = \frac{m_A}{m_A + m_B} \frac{v_0^2}{2g} < \frac{m_A l}{m_A + m_B} < l$$

3. The rolling cylinder has instantaneous center located at A, and its angular velocity and acceleration are given by

$$\omega = \frac{v_O}{r}$$

$$\alpha = \dot{\omega} = 0$$

For two points A and O on the rolling cylinder,

$$\mathbf{a}_A = \mathbf{a}_O + \boldsymbol{\omega}_{AO} \times (\boldsymbol{\omega}_{AO} \times \mathbf{r}_{A/O}) + \boldsymbol{\alpha}_{AO} \times \mathbf{r}_{A/O}$$

$$= -\frac{v_O^2}{R+r}\mathbf{j} + (-\omega\mathbf{k}) \times [(-\omega\mathbf{k}) \times (-r\mathbf{j})] = -\frac{v_O^2}{R+r}\mathbf{j} + r\omega^2\mathbf{j}$$

$$= \frac{R}{r(R+r)} v_O^2 \mathbf{j} \qquad \uparrow$$

Between A and B on the rolling cylinder,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\omega}_{BA} \times (\boldsymbol{\omega}_{BA} \times \mathbf{r}_{B/A}) + \boldsymbol{\alpha}_{BA} \times \mathbf{r}_{B/A}$$

$$= \frac{Rv_O^2}{r(R+r)}\mathbf{j} + (-\omega\mathbf{k}) \times [(-\omega\mathbf{k}) \times (2r\mathbf{j})] = \frac{Rv_O^2}{r(R+r)}\mathbf{j} - 2r\omega^2\mathbf{j}$$

$$= -\frac{R+2r}{r(R+r)} v_O^2 \mathbf{j} \qquad \downarrow$$