Department of Physics
University of California, Berkeley
Physics 7b Section 2
Spring semester 2007
Mid-term examination 1
Tuesday Feb. 20, 6:00 – 8:00 PM

Solutions

1) (15 points) A cabin in the sierra mountains has walls, floor and ceiling of total area 200 m^2 covered with insulation having an R value of 1.0 m^2C°/W ($R \equiv L/K$, K = thermal conductivity). The cabin is heated by a wood-burning stove. The temperature outside on a winter night is -20 C° while the temperature in the cabin is 20 C°.
   a) Assuming the walls, floor and ceiling all lose heat equally, how much heat/second is lost from the cabin?

The cabin heat is lost by conduction through the walls.

$dQ/dt = -KA/L\ \Delta T = -A/R\ \Delta T = -200\ m^2/\ 1.0\ m^2C°/W \cdot 40\ C° = -8000\ W$

   b) The stove provides half its heat through thermal radiation and the other half through conduction+convection. If the stove has an outer surface area of 2 m^2 and is painted black (emissivity = 0.9) what temperature does it have to keep the interior cabin temperature constant?

The stove must provide all the heat lost through the walls to maintain a steady interior temperature. Radiation accounts for half the heat provided.

$dQ/dt\,|_{radiation} = \varepsilon\sigma A_{stove} T_{stove}^4 = 0.5 \cdot dQ/dt\,|_{lost}$,

$T_{stove}^4 = 0.5 \cdot dQ/dt\,|_{lost}/\varepsilon\sigma A_{stove} \qquad = 0.5 \cdot 8000W/0.9 \cdot (5.67\times10^{-8}\ W/m^2K^4) \cdot 2m^2$

$= 3.92 \times 10^{10}\ K^4$

$T_{stove} = (3.92 \times 10^{10})^{1/4}\ K = 445\ K$

c) When burned, the wood provides 1720 kcal/kg. If 1 log of wood weighs 3 kg, how many logs/hour are needed to keep the stove at the required temperature?

The amount of heat provided by the stove in 1 hour must equal the heat content of the logs it burned.

$Q_{1\ hour} = dQ/dt \cdot \text{seconds/hour} = \text{logs/hour} \cdot \text{mass/log} \cdot Q_{wood}/\text{mass}$

$\text{Logs/hour} = dQ/dt \cdot \text{seconds/hour} / (\text{mass/log} \cdot Q_{wood}/\text{mass})$

$= 8000\ W \cdot 3600\ \text{seconds/hour} / (3\text{kg} \cdot 1720\ \text{kcal/kg} \cdot 4186\ \text{J/kcal})$

$= 4/3$

2) (15 points) A large SUV of mass M_{SUV} is traveling at V_{SUV} when it must stop suddenly due to a traffic jam. Its brakes consist of 4 steel disks (rotors) weighing m_{disk} each.

a) Assuming all the SUVs kinetic energy is absorbed by its brake disks, equally by each disk, by how much will their temperature increase? Evaluate your result taking M_{SUV} = 2000 kg, V_{SUV} = 100 km/hour, and m_{disk} = 5kg.

The change in internal energy of the brakes must equal the amount of heat added as there is no work. The heat added must equal the kinetic energy lost by the SUV.

$\Delta U = m_{brakes}C_{steel}\Delta T_{brakes} = Q = 0.5\ M_{SUV}\ V^2_{SUV}$

$\Delta T_{brakes} = 0.5\ M_{SUV}\ V^2_{SUV}/m_{brakes} \cdot C_{steel}$

$= 0.5 \cdot 2000\ \text{kg} \cdot (10^5\ \text{m/hour} \cdot 1/3600\ \text{hours/sec})^2 / (4 \cdot 5\text{kg} \cdot 450\ \text{J/kg} \cdot \text{C}°)$

$= 86\ \text{C}°$

b) If the disks initially have diameter 40 cm, by what amount will their diameter increase due to this temperature change?

The disk diameter is a linear measure. The volume expansion coefficient is 3 times the linear expansion coefficient.

$\alpha_{steel} = 1/3\ \beta_{steel} = 1/3\ \ 3.5\times10^{-5}/\text{C}° = 1.2\times10^{-5}/\text{C}°$

$\Delta D = D\ \alpha_{steel}\ \Delta T_{brakes} = 40\ \text{cm} \cdot 1.2\times10^{-5}/\text{C}° \cdot 86\ \text{C}° = 0.04\ \text{cm}$

c) Assuming the initial temperature of the brakes was 20 C°, by how much does the entropy of the entire SUV change in coming to a stop?

Only the brakes of the SUV experience an entropy change. A reversible process that gives the same final state is if heat is slowly added from an (infinite) succession of heat reservoirs without changing the reservoirs temperature.

T_i = 20 C° = 293 K

$T_f = T_i + \Delta T$ = 293 + 86 K = 379 K

$\Delta S = \int_{Ti}^{Tf} dQ/T = \int_{Ti}^{Tf} mC\, dT/T = mC \ln(T_f/T_i)$

$= 4 \cdot 5\text{kg} \cdot 450\ \text{J/kg}\cdot\text{K} \ln(379/293) = 2316\ \text{J/K}$

3) (15 points) A real heat engine working between heat reservoirs at temperatures T_h = 600 K and T_l = 300 K produces 400 J of work per cycle for a heat input of 1000 J.

a) Compare the efficiency of this real engine to that of a Carnot engine operating between the same temperatures

The efficiency is defined as the work done divided by the heat provided at T_h.

$\varepsilon = W/Q_h = 400\ \text{J}/1000\ \text{J} = 0.4$

The Carnot efficiency is given in terms of the high and low temperatures.

$\varepsilon_C = 1 - T_l/T_h = 1 - 300\ \text{K}/600\ \text{K} = 0.5$, higher than the real efficiency

b) Calculate the total entropy change of the universe (engine + environment) for one cycle of the real engine

The engine does not change entropy in a cycle. The environment changes entropy due to heat transfer from the high-temperature reservoir and to the low-temperature reservoir. The reservoirs do not change temperature.

$\Delta S = \Delta S_h + \Delta S_l = -Q_h/T_h + Q_l/T_l$

$= -1000\text{J}/600\ \text{K} + 600\text{J}/300\ \text{K} = 1/3\ \text{J/K}$

c) Calculate the total entropy change of the universe for a Carnot engine operating between the same temperatures

The same formula for the entropy change as above applies. The amount of heat exhausted to the low temperature reservoir is given by energy conservation for 1 cycle. The Work done is given by the efficiency.

$W = \varepsilon_C Q_h$, $\Delta U_{cycle} = 0 = Q - W = Q_h - Q_l - W = Q_h(1 - \varepsilon_C) - Q_l$

$Q_l = Q_h(1- \varepsilon_C) = Q_h(1 - (1 - T_l/T_h)) = Q_h \cdot T_l/T_h$

$\Delta S = \Delta S_h + \Delta S_l = -Q_h/T_h + Q_l/T_l = -Q_h/T_h + (Q_h \cdot T_l/T_h)/T_l$

$= -Q_h/T_h + Q_h/T_h = 0$

d) Show that the difference in work done by these two engines per cycle is $T_l \, \Delta S$, where T_l is the temperature of the low-temperature reservoir (300 K) and ΔS is the entropy increase per cycle of the real engine.

$\Delta W = W_C - W_{real} = Q_h \cdot \varepsilon_C - W_{real} = 1000\text{J} \cdot 0.5 - 400\text{J} = 100\text{J}$

$\Delta S \cdot T_l = 1/3 \text{ J/K} \cdot 300 \text{ K} = 100\text{J}$

4) (15 points) An ice maker takes in water through a pipe at room temperature T_r and produces ice at $T_i < 0$ C°. When running steadily the ice maker consumes power P and can produce M_i mass of ice in 1 hour. The heat generated from the refrigeration unit of the ice maker is exhausted into the room.

a) How much heat is exhausted by the ice maker in 1 hour of steady running? Evaluate your result using T_r=20 C°, T_i = -10 C°, P = 200 W, and M_i = 5kg.

The heat exhausted is the heat extracted from the water in making ice plus the work done in 1 hour, which is equal to the power times 1 hour. The heat extracted to make ice is the heat needed to cool the water to 0C° plus the heat to change the water to ice at 0C° plus the heat to cool the ice.

$Q_{extracted} = Q_{water} + Q_{change} + Q_{ice}$

$= M_iC_w(T_r - 0) + M_iQ_{fusion} + M_iC_w(0 - T_i)$

$= M_i(C_wT_r + Q_{fusion} - C_wT_i)$

$= 5\text{kg} \, (1 \text{ kcal/kg} \cdot 20 \text{ C}° + 80 \text{ kcal/kg} + 0.5 \text{ kcal/kg} \cdot 10 \text{ C}°) = 525 \text{ kcal}$

$W = P \cdot (1 \text{ hour}) = 200 \text{ J/sec} \cdot 3600\text{sec/hour} / 4186 \text{ kcal/J} = 172 \text{ kcal}$

$Q_{exhaust} = Q_{extracted} + W = 525 \text{ kcal} + 172 \text{ kcal} = 697 \text{ kcal}$

b) Using the above values, what is the Coefficient of Performance (CP) of the refrigeration unit? Compare that to the ideal CP for these temperatures.

The Coefficient of Performance of a refrigerator is the heat extracted divided by the work. The ideal CP is given by the room (high) and ice (low) temperatures.

$CP = Q_{extracted}/W = 525 \text{ kcal}/172 \text{ kcal} = 3.05$

$CP_{ideal} = T_i/(T_r - T_i) = (273 - 10) \text{ K}/30 \text{ K} = 8.77$

c) Assuming the actual CP scales as the ideal (ie that CP_{actual}/CP_{ideal} = constant), how much extra power is needed to generate ice at the same rate if the room temperature increases 5 C°?

Let (') denote values at the increased temperature. Then we have

$CP'_{ideal} = T_i/(T_r+5 - T_i) = 263\ K/35\ K = 7.5$

$CP'_{actual}/CP'_{ideal} = CP_{actual}/CP_{ideal}$, or re-arranging

$CP'_{actual} = CP_{actual} \cdot CP'_{ideal}/CP_{ideal} = CP_{actual} \cdot 7.5/8.77 = CP_{actual} \cdot 0.855$

The actual efficiency is also given by the standard formula

$CP'_{actual} = Q_{extracted}/W'$

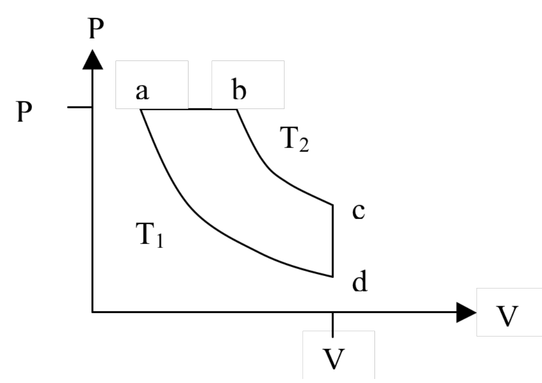
$W' = Q_{extracted}/CP'_{actual} = Q_{extracted}/CP_{actual} \cdot 0.855 = W / 0.855$

The Power is the work/unit time and so scales with the Work

$P' = P/0.855 = 200\ W/0.855 = 234W$

$\Delta P = P' - P = 34W$

5) (20 points) A system of n moles of ideal gas with constant-volume molar specific heat C_V is made to undergo a cycle (see figure below) with the following stages: (a→b) an isobaric expansion at pressure P, (b→c) an isothermal expansion at temperature T_2, (c→d) an isochoric depressurization at volume V, and (d→a) an



isothermal compression at temperature T_1.

a) What are the change in internal energy of the gas ΔU, the work done by the gas W, and the heat added to the gas Q for each stage of this cycle? Express the results using T_1, T_2, and $T_3 \equiv PV/nR$.

From the ideal gas law PV = nRT we get $V_a = nRT_1/P$, $V_b = nRT_2/P$

From a→b is an isobaric transition

$\Delta U = n\, C_V(T_2 - T_1)$

$W = P(V_b - V_a) = P(nRT_2/P - nRT_1/P) = nR(T_2 - T_1)$

$\Delta U = Q - W$, so

$Q = \Delta U + W = n(R + C_V)(T_2 - T_1) = nC_P(T_2 - T_1)$

From b→c is an isothermal transition
$\Delta U = 0 = Q - W$

$Q = W = \int_{V_b}^{V_c} P\, dV = nRT_2 \ln(V_c/V_b)$

$V_c = V$, and taking V_b from above gives

$W = nRT_2\ln(VP/nRT_2) = nRT_2\ln(nRT_3/nRT_2) = nRT_2\ln(T_3/T_2)$

From c→d is an isochoric transition
$\Delta V = 0 \Rightarrow W = 0$

$\Delta U = Q = nC_V(T_1 - T_2)$

From d→a is an isothermal transition
$\Delta U = 0$

$Q = W = nRT_1\ln(V_a/V_d)$

$V_d = V$, and taking V_a from above gives

$W = nRT_1\ln(nRT_1/VP) = nRT_1\ln(nRT_1/nRT_3) = nRT_1\ln(T_1/T_3)$

b) What is the efficiency of this cycle?

The efficiency is given in terms of the heat lost at low temperature and the heat gained at high temperature.

$\varepsilon = 1 - |Q_l| / |Q_h|$

$Q_l = Q_{cd} + Q_{da} = nC_V(T_1 - T_2) + nRT_1\ln(T_1/T_3)$

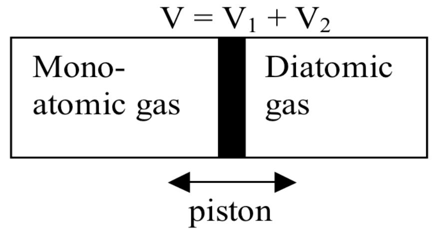
$Q_h = Q_{ab} + Q_{bc} = nC_P(T_2 - T_1) + nRT_2\ln(T_3/T_2)$

$$\varepsilon = 1 - (C_V(T_2 - T_1) + RT_1\ln(T_3/T_1)) / (C_P(T_2 - T_1) + RT_2\ln(T_3/T_2))$$

$$= R((T_2 - T_1) + T_2\ln(T_3/T_2) - T_1\ln(T_3/T_1)) / (C_P(T_2 - T_1) + RT_2\ln(T_3/T_2))$$

6) (20 points) A sealed, insulated cylindrical container of volume V contains 1 mole each of monoatomic and diatomic ideal gases. The two gases are separated into opposite ends of the cylinder by a heat-conducting piston that is free to move (see figure). Initially the monoatomic gas has temperature T_m K, the diatomic gas has

temperature T_d K, with $T_d < T_m$. The system is then allowed to reach thermal equilibrium.



a) What fraction of the volume V is occupied by the different gases initially?

The pressure of the gases must be equal, as otherwise the piston would move.

We also have the ideal gas law PV = nRT, or P = nRT/V

$P_d = nRT_d/V_d = P_m = nRT_m/V_m$, or $T_d/V_d = T_m/V_m$, or $T_dV_m = T_mV_d$

We also have

$V_d + V_m = V$, or $V_d = V - V_m$. Putting these together gives

$T_dV_m = T_m(V – V_m)$. Solving for V_m gives:

$V_m/V = T_m/(T_d + T_m)$. Plugging this into the expression for V gives

$V_d/V = 1 -T_m/(T_d + T_m) = T_d/(T_d + T_m)$

b) If we neglect the heat capacity of the piston, what is the final temperature of the gases at thermal equilibrium?

At thermal equilibrium the temperatures of the two gases must be equal. Total energy must also be conserved.

T_f = same for both gases

$\Delta U = 0 = \Delta U_d + \Delta U_m = n\,C_d(T_f – T_d) + n\,C_m(T_f – T_m)$. Solving for T_f gives:

$T_f = (C_dT_d + C_mT_m)/(C_d + C_m)$

We also know $C_m = 3/2$ R, $C_d = 5/2$ R, so

$T_f = (5T_d + 3T_m)/8$

c) What fraction of the volume V is occupied by the different gases at thermal equilibrium?

The same formula as we found in part a still applies, just at the new temperature.

$V_m/V = T_m/(T_d + T_m) = T_f/(T_f + T_f) = 0.5$

$V_d/V = 0.5$

d) What is the change in entropy of each gas going from the initial condition to thermal equilibrium? (hint: use a reversible path between the same initial and final states to compute ΔS).

Both pressure and volume change in reaching equilibrium. The entropy change can be calculated using an equivalent reversible process, for instance an isochoric followed by an isobaric transition.

Isobaric:

$\Delta S = \int_{T_i}^{T_f} dQ/T = \int_{T_i}^{T_f} n\, C_P dT/T = nC_P \ln(T_f/T_i) = nC_P \ln(V_f/V_i)$

Isochoric:

$\Delta S = \int_{T_i}^{T_f} dQ/T = \int_{T_i}^{T_f} n\, C_V\, dT/T = nC_V \ln(T_f/T_i) = nC_V \ln(P_f/P_i)$, where

$P_f = nRT_f/V_f = 2nRT_f/V$

$P_i = nRT_m/V_m = nR(T_d + T_m)/V$

Adding these for the different gases and using the values found above gives

$\Delta S_m = nR(5/2 \ln((T_d+T_m)/2\cdot T_m) + 3/2 \ln((5T_d + 3T_m)/4(T_d + T_m)))$

$\Delta S_d = nR(7/2 \ln((T_d+T_m)/2\cdot T_d) + 5/2 \ln((5T_d + 3T_m)/4(T_d + T_m)))$