STATISTICS 134 | | S.M. STOYANOV

FALL 2011

MIDTERM 2

NAME: ______________________ SID: ______________________

Please show ALL WORK AND REASONING for ALL the problems. Unless indicated otherwise, please work the problem through to a numerical answer. You may use a calculator and a handwritten page of notes.

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1. Let μ denote the true average caffeine content in each 12 oz. can of a certain cola. A sample of size n cans of this cola is collected. The amount of caffeine in each is a random variable with mean μ and standard deviation 0.1 mg. How large should n be in order to ensure that the probability that sample average caffeine content is within 0.02 mg of the true average caffeine content is at least 0.95. [/3]

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2. Let X be normally distributed with mean 80 and standard deviation 2. Find a constant c such that $P(|X - 80| > c) \approx 1/3$. [/3]

3. Let X be a discrete random variable with moment generating function $M_X(t) = .3 + .2e^{2.5t} + .5e^{3t}$.
Find the probability distribution of X, and $Var(X)$. [/4]

4. Let Z be a standard normal random variable (with mean 0 and variance 1). Let X be Bernoulli, taking values 1 and -1 with equal probability, and let $Y = XZ$. Is Y standard normal as well? Explain your answer by either showing that it must be, or that it cannot be. [/4]

5. If X and Y are jointly distributed discrete random variables with $P(X = i, Y = j) = \frac{ij}{18}$ for $i = 1, 2$ and $j = 1, 2, 3$, find

(a) $P(X + Y > 3)$.

[/3]

(b) The marginal distribution of Y.

[/3]

6. Let the discrete random variables U_1, U_2 be independent, and uniform on $\{1, 2, \ldots, n\}$, (so $P(U_i = k) = 1/n$), and let $M = \max(U_1, U_2)$. Find the distribution of M. [/3]

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7. Suppose that X has Poisson (μ) distribution, and Y has geometric (p) distribution on $\{0, 1, 2, \ldots\}$ independently of X. Find a formula for $P(Y \geq X)$ in terms of μ and p, and evaluate it numerically for $p = 1/2$, $\mu = 1$. [/4]