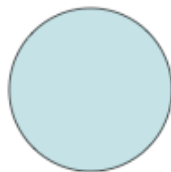
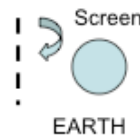


**Note:** This is a closed book exam. One sheet of notes (both sides) is allowed.  
Do all 4 problems. Please display your answers clearly (e.g., in a box) and cross out any work that you do not wish to be graded.

- (25) 1. Answer the following short questions using a few sentences and/or appropriate equations, if needed.
- (8) a) Two systems, A and B, are in thermal and diffusive contact. What are the conditions on the entropies ( $\sigma_A$  and  $\sigma_B$ ) and free energies ( $F_A$  and  $F_B$ ) for the two systems in equilibrium?
  - (8) b) Suppose that a new star is discovered nearby. Describe briefly how you can estimate the surface temperature of this star by measuring the spectrum of its emitted light, using what you have learned about the black body radiation spectrum.
  - (9) c) A gas of  $10^{23}$  argon atoms (assumed to be ideal) is compressed quasistatically from 20 liters to 5 liters at a constant temperature of  $250^\circ$  K. What is the change in entropy of the gas?
- (25) 2. Thermal radiation
- (12) a) Let us assume that the sun and the earth are perfect black-body radiators and that there is no source of energy other than the sun. Show that at steady-state, the temperature on the earth's surface is linearly proportional to the sun's surface temperature.
  - (13) b) To reduce global warming, it has been suggested recently that one can place a giant solar screen (at the so-called Lagrange point) between the sun and the earth to reduce the solar flux arriving on earth. Suppose that one is able to construct such a screen to reduce the solar flux to earth by 5%. How much (in  $^\circ$  K) will the average temperature of the earth be reduced by? Assume a value for the earth's temperature without the screen to be  $280^\circ$  K.



SUN



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- (25) 3. Consider a system of  $N$  harmonic oscillators on a lattice. Assume that the oscillators are independent of each other and that the quantum states of each oscillator are labeled by a non-negative integer  $n$  with energy

$$E = n\hbar\omega, \quad n = 0, 1, 2, \dots, \infty$$

where  $\omega$  is the same frequency for each oscillator.

- (10) a) Find the partition function  $Z(\tau)$ .

- (8) b) Calculate the mean total energy  $U(\tau)$  of the system.

- (7) c) Calculate its heat capacity  $C_V = (\partial U / \partial \tau)_V$  in the limit  $\tau \rightarrow 0$ . How is this result different from that of the Debye model?

- (25) 4. Consider the following problem of adsorbates on a surface. The surface is at thermal and diffusive equilibrium with a gas of the adsorbate atoms. We may assume that the adsorption sites are independent from each other. Each site may be unoccupied with energy zero or occupied by one particle in either of two states, a or b, both of energy  $\epsilon_a = \epsilon_b = -E$ .

- (10) a) Focusing on a single site, find the probability of finding a particle in the quantum state a.

- (10) b) Show that the mean occupation for a site is

$$f(\mu, \tau) = \frac{2}{e^{-(\mu+E)/\tau} + 2}$$

where  $\mu$  = chemical potential and  $\tau$  = temperature.

- (5) c) Express the above result for  $f$  in terms of the pressure  $p$  and temperature  $\tau$  of the gas of adsorbate atoms. You may assume the gas to be a classical ideal gas.

**END OF EXAM**