CS 70 Second Midterm 7 April 2011

NAME (1 pt): ____________________

SID (1 pt): ____________________

TA (1 pt): ____________________

Name of Neighbor to your left (1 pt): ____________________

Name of Neighbor to your right (1 pt): ____________________

Instructions: This is a closed book, closed calculator, closed computer, closed network, open brain exam, but you are permitted a 1 page, double-sided set of notes, large enough to read without a magnifying glass.

You get one point each for filling in the 5 lines at the top of this page. Each other question is worth 20 points.

Write all your answers on this exam. If you need scratch paper, ask for it, write your name on each sheet, and attach it when you turn it in (we have a stapler).

1	
2	
3	
4	
5	
Total	

Question 1 (20 points). For full credit explain your answers.

Part 1 (10 points). Pick a random integer n in the range from 0 to 9,999,999, each with equal probability. What is the probability that the decimal digits of n add up to 9? Fill in your answer in the box below.

P=

Part 2 (5 points). What is the probability that the decimal digits of n add up to 10? Fill in your answer in the box below.

P=

Part 3 (5 points). What is the probability that the decimal digits of n add up to 11? Fill in your answer in the box below.

P=

Question 1 (20 points). For full credit explain your answers.

Part 1 (10 points). Pick a random integer n in the range from 0 to 99,999,999, each with equal probability. What is the probability that the decimal digits of n add up to 9? Fill in your answer in the box below.

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P=

Part 3 (5 points). What is the probability that the decimal digits of n add up to 11? Fill in your answer in the box below.

P=

Question 2 (20 points). For full credit explain your answers.

1. (*10 points*)

   Let the sample space $\Omega = \{0, 1, 2, 3\}$, and let the probability of each sample point be uniform. What is the probability of the events $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{1, 3\}$? Are events A and B independent? What is $Pr[A|B \cup C]$?

2. (*10 points*)

   Suppose you are given a bag containing n unbiased coins. You are told that $n-1$ of these are normal coins, with heads on one side and tails on the other; however, the remaining coin has heads on both its sides.

   - Suppose you reach into the bag, pick out a coin uniformly at random, flip it and get a head. What is the (conditional) probability that this coin you chose is the fake (i.e., double-headed) coin?

   - Suppose you flip the coin k times after picking it (instead of just once) and see k heads. What is now the conditional probability that you picked the fake coin?

Question 2 (20 points). For full credit explain your answers.

1. (*10 points*)

   Let the sample space $S = \{a, b, c, d\}$, and let the probability of each sample point be uniform. What is the probability of the events $A = \{b, c\}$, $B = \{c, d\}$, $C = \{b, d\}$? Are events A and B independent? What is $Pr[A|B \cup C]$?

2. (*10 points*)

   Suppose you are given a bag containing m unbiased coins. You are told that $m - 1$ of these are normal coins, with heads on one side and tails on the other; however, the remaining coin has tails on both its sides.

   - Suppose you reach into the bag, pick out a coin uniformly at random, flip it and get a tail. What is the (conditional) probability that this coin you chose is the fake (i.e., double-tailed) coin?

   - Suppose you flip the coin s times after picking it (instead of just once) and see s tails. What is now the conditional probability that you picked the fake coin?

Question 3 (20 points) Bayes' Casino.

At Bayes' Casino in Las Vegas, there are two types of slot machines: Red and Blue. Every machine of one color results in a 'win' 10% of the time, and every machine of the other color results in a win 25% of the time. (A 'win' is when the machine returns money). Nobody knows which color wins more frequently, but you are 80% sure it's the Blue machines. You find a Blue machine in the casino and play a quarter.

(**a**) 6 points. Let \$ be the event the Blue machine wins. Let A be the event that the Blue machine is a "good" (25%) one. Write down the following probabilities:

(**b**) 7 points. Suppose the Blue machine does not win. Given this event and your 80% initial estimate, what is the probability that the Blue machines have the better win rate (25%)? Feel free to write your answer as a fraction. Show your work in order to earn any partial credit.

(**c**) 7 points. Repeat part (b), supposing instead that the Blue machine wins.

Question 3 (20 points) Bayes' Casino.

At Bayes' Casino in Las Vegas, there are two types of slot machines: Red and Blue. Every machine of one color results in a 'win' 15% of the time, and every machine of the other color results in a win 20% of the time. (A 'win' is when the machine returns money). Nobody knows which color wins more frequently, but you are 75% sure it's the Blue machines. You find a Blue machine in the casino and play a quarter.

(**a**) 6 points. Let \$ be the event the Blue machine wins. Let A be the event that the Blue machine is a "good" (20%) one. Write down the following probabilities:

(**b**) 7 points. Suppose the Blue machine does not win. Given this event and your 75% initial estimate, what is the probability that the Blue machines have the better win rate (20%)? Feel free to write your answer as a fraction. Show your work in order to earn any partial credit.

(**c**) 7 points. Repeat part (b), supposing instead that the Blue machine wins.

Question 4 (20 points) Binomial Distribution.

4.1 (10 points). Suppose X is a random variable that can take positive integer values $0, 1, \ldots, m$ and β is some real number such that $0 \le \beta \le 1$. Distribution of X is given by the following recurrence relation:

$$P(X = k) = \begin{cases} (1-\beta)^m & \text{for } k = 0 \\ \frac{\beta}{1-\beta} \cdot \frac{m-k+1}{k} \cdot P(X = k-1) & \text{for } k = 1, 2, \ldots, m \end{cases}$$

Use induction to prove that X is actually a binomially distributed random variable. What are the parameters of the binomial distribution? What is $E(X)$?

4.2 (10 points). You have two boxes A and B, each containing n balls. You randomly pick one box and then take one ball out of it. You continue this process until you pick a box and find it empty. Suppose X is the number of balls that remain in the other box when you stop. If probability of picking A and B are p and $1-p$ respectively, write down the distribution of X in terms of n and p.

Question 4 (20 points) Binomial Distribution.

4.1 (10 points). Suppose X is a random variable that can take positive integer values $0, 1, \ldots, n$ and μ is some real number such that $0 \leq \mu \leq 1$. Distribution of X is given by the following recurrence relation:

$$P(X = k) = \begin{cases} (1-\mu)^n & \text{for } k = 0 \\ \frac{\mu}{1-\mu} \cdot \frac{n-k+1}{k} \cdot P(X = k-1) & \text{for } k = 1, 2, \ldots, n \end{cases}$$

Use induction to prove that X is actually a binomially distributed random variable. What are the parameters of the binomial distribution? What is $E(X)$?

4.2 (10 points). You have two boxes R and S, each containing m balls. You randomly pick one box and then take one ball out of it. You continue this process until you pick a box and find it empty. Suppose Y is the number of balls that remain in the other box when you stop. If probability of picking R and S are q and $1-q$ respectively, write down the distribution of Y in terms of m and q.

Question 5 (20 points) Random Variables. Instead of a pair of the usual 6-sided dice, you can play a game with one 4 sided die (sides numbered 1 through 4, each equally likely to come up), and one 8 sided die (sides numbered 1 through 8, again all equally likely). Let A be the sum of the values that come up on these two dice.

5.1 (5 points) What is the expected value of A?

$E(A) =$

5.2 (5 points) What is the probability $A = 8$?

$P(A = 8) =$

5.3 (10 points) You play a friendly betting game with your friend where you roll two dice each round and if the sum of the two dice is 7 your friend pays you \$5, otherwise you pay your friend \$1. If you play this game with two 6-sided dice your expected profit is \$0 each round. Define a random variable and use it to calculate the expected amount of money you win or lose in 1 round if you play using a 4-sided die and an 8-sided die. In the box below specify whether you win or lose money (by circling the appropriate word) and fill in the expected amount of money you win or lose in one round.

In one round you expect to **win/lose (circle one)** \$

Question 5 (20 points) Random Variables. Instead of a pair of the usual 6-sided dice, you can play a game with one 4 sided die (sides numbered 1 through 4, each equally likely to come up), and one 8 sided die (sides numbered 1 through 8, again all equally likely). Let S be the sum of the values that come up on these two dice.

5.1 (5 points) What is the expected value of S?

$E(S) =$

5.2 (5 points) What is the probability $S = 8$?

$P(S = 8) =$

5.3 (10 points) You play a friendly betting game with your friend where you roll two dice each round and if the sum of the two dice is 7 your friend pays you \$5, otherwise you pay your friend \$1. If you play this game with two 6-sided dice your expected profit is \$0 each round. Define a random variable and use it to calculate the expected amount of money you win or lose in 1 round if you play using a 4-sided die and an 8-sided die. In the box below specify whether you win or lose money (by circling the appropriate word) and fill in the expected amount of money you win or lose in one round.

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