1) Natural Gas

$$\left(\frac{\$0.67}{1\ \cancel{\text{therm}}}\right)\left(\frac{1\ \cancel{\text{therm}}}{10^5\ \cancel{\text{BTU}}}\right)\left(\frac{1\ \cancel{\text{BTU}}}{1057\ \text{J}}\right) = 6.3\times10^{-9}\ \text{dollars}/\text{Joule}$$

$$Q = mc\Delta T$$

$$1\ \text{BTU} = (1\ \text{lb})\,c\,(1\,^\circ\text{F}) = \left(\frac{1}{2.2}\ \text{kg}\right)\left(4186\ \frac{\text{J}}{\text{kg}\,^\circ\text{C}}\right)\left(\frac{5}{9}\ \text{C}^\circ\right) \approx 1057\ \text{J}$$

Electricity

$$\left(\frac{\$0.14}{\text{kW}\cdot\text{hr}}\right)\left(\frac{1\ \text{kW}}{1000\ \text{W}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{sec}}\right) = 3.9\times10^{-8}\ \frac{\text{dollars}}{\text{Joule}}$$

Watts · secs = Joules

Ratio of $\dfrac{\text{Natural Gas cost}}{\text{Electricity cost}} = .16$

Problem 2 Solution

a). Since the air is an ideal gas we can use the ideal gas law.

$$PV = nRT = NkT$$

$$\Rightarrow N = \frac{PV}{kT} = \frac{P_0 V_0}{k T_a} = \frac{(150 \times 1.013\times10^5 \text{ N/m}^2)(12\times10^{-3} \text{ m}^3)}{(1.38\times10^{-23} \text{ J/K})(293 \text{ K})}$$

$$\approx 4.51 \times 10^{25} \text{ (molecules)}$$

b). The fractional Volume change of the tank

$$= \frac{\Delta V_{tank}}{V_0} = \frac{V_0 (3\alpha)\Delta T}{V_0} \quad (\because \beta = 3\alpha)$$

$$= 3\alpha \Delta T$$

$$= 3(2.5\times10^{-5} (C^\circ)^{-1})(-20 C^\circ)$$

$$= -0.0015 = -0.15\%$$

c). At depth $h = 20\text{m}$, $T = T_a - 20C^\circ = 0\ C^\circ$

$$P = P_{atm} + \rho_w g h$$

For each breath, the diver draws in

$$n = \frac{PV}{RT} = \frac{(P_{atm} + \rho_w g h)V}{RT}$$

the available air in the tank for the diver to breath is

$$n' = n_{initial} - n_{final}$$

$$n' = \frac{P_0 V_0}{R T_a} - \frac{(P_{atm} + \rho_w g h) V'}{R T}$$

where $V' = (1 - 0.15\%) V_0$

Finally,

Time $$t = \frac{n'}{n f} = \frac{\frac{P_0 V_0}{R T_a} - \frac{(P_{atm} + \rho_w g h) V'}{R T}}{\frac{(P_{atm} + \rho_w g h) V}{R T} \cdot f}$$

$$= \frac{P_0 V_0}{(P_{atm} + \rho_w g h) V f} \cdot \frac{T}{T_a} - \frac{V'}{V f}$$

$$= \frac{(150 \times 1.013 \times 10^5 \text{ N/m}^2)(12 \times 10^{-3} \text{ m}^3)}{[1.013 \times 10^5 \text{ N/m}^2 + (1.00 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(20 \text{ m})](1.5 \times 10^{-3} \text{ m}^3)(11)} \cdot \frac{273 \text{K}}{293 \text{K}}$$

$$- \frac{(1 - 0.15\%)(12 \times 10^{-3} \text{ m}^3)}{(1.5 \times 10^{-3} \text{m}^3)(11)}$$

≈ 34 minutes

d) $$n_{bubble} = \frac{P V_{bubble}}{R T} = \frac{P \frac{4}{3} \pi (\frac{D}{2})^3}{R T} = \frac{\pi P D^3}{6 R T}$$

$$E_{int} = \frac{5}{2} n R T$$

$$= \frac{5}{2} \frac{\pi P D^3}{6 R T} \cdot R T$$

$$= \frac{5}{12} \pi P D^3$$

$$= \frac{5}{12} \pi (P_{atm} + \rho_w g h) D^3$$

$$= \frac{5}{12} \pi [1.013 \times 10^5 \text{ N/m}^2 + (1.00 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(20 \text{m})] (3 \times 10^{-2} \text{ m})^3$$

≈ 10.5 J

e). At the depth $h = 20$ m,

$$n_{bubble} = \frac{P V_{bubble}}{RT}$$

At the surface.

$$n'_{bubble} = \frac{P_{atm} V'_{bubble}}{R T_a}$$

$$n_{bubble} = n'_{bubble} \Rightarrow \frac{P V_{bubble}}{RT} = \frac{P_{atm} V'_{bubble}}{R T_a}$$

$$V'_{bubble} = \frac{T_a}{T} \cdot \frac{P}{P_{atm}} \cdot V_{bubble}$$

$$\frac{4}{3}\pi\left(\frac{D'}{2}\right)^3 = \frac{T_a}{T} \frac{P}{P_{atm}} \left(\frac{D}{2}\right)^3 \cdot \frac{4}{3}\pi$$

$$D' = \left(\frac{T_a}{T} \frac{P}{P_{atm}}\right)^{\frac{1}{3}} \cdot D$$

$$= \left[\frac{T_a}{T} \cdot \frac{(P_{atm} + \rho_w g h)}{P_{atm}}\right]^{\frac{1}{3}} \cdot D$$

$$= \left[\frac{293\ k}{273\ K} \cdot \left(1 + \frac{(1.00 \times 10^3\ kg/m^3)(9.81\ m/s^2)(20m)}{1.013 \times 10^5\ N/m}\right)\right]^{\frac{1}{3}} \cdot (3 \times 10^{-2}\ m)$$

$$\approx 4.4 \times 10^{-2}\ m$$

f). $V_{rms} = \sqrt{\frac{3RT_a}{M}} = \sqrt{\frac{3(8.315\ J/mol \cdot k) \cdot 293\ K}{32 \times 10^{-3}\ kg/mol}} \approx 477.9\ m/s$

g). emissivity $e=1$, it is an ideal Black Body. Its surface does not reflect. It absorbs all incident energy and re-emit it as infrared energy.

.
Hannah: Is there anything in it?
Valentine: In what? We are all doomed? (*casually*) Oh yes, sure - it's called the second law of thermodynamics.
Hannah: Was it known about?
Valentine: By poets and lunatics from time immemorial
-*Arcadia*, by Tom Stoppard

3: Calorimetry and Entropy (30 Points)

A hot block of metal with inital temperature T_m, mass M, and specific heat C_m is dropped from a height h into a container of water, specific heat $C_w = 2C_m$, which also comains mass M of water but is at initial temperature T_w. Find the entropy change of the system (metal plus water) after the block has come to rest in the water and the combined system has reached thermal equilibrium. You may assume that no water is vaporized. Neglect the container wall heat capacity and air resistance.

Before we calculate any entropy changes, we need to find the final temperature of the equilibrated system. In most calorimetry problems, we would just apply $\sum Q = 0$. There is a subtlety here, however. When we say that the heats of the components sum to 0, we are really saying that no heat enters or leaves the system. That is *not* the case here, however. We are adding heat since we are dropping the block in. Initially, the metal has a potential energy Mgh, which is all converted to kinetic energy as it reaches the end of its fall. What happens to this energy? It becomes additional heat added to the system! Using our first law, $\Delta E = \Delta Q - \Delta W$ and noting that there is no work being done, we can conclude that $Q_{drop} = Mgh$. We add this to the *right* side of the equation. We do this because the equation will then tell us that the heat gained by the water minus the heat lost by the metal is equal to the additional heat added to the system.

For the heats lost by the metal and gained by the water, we merely use $\Delta Q = MC\Delta T$ where, as always, $\Delta T = (T_f - T_i)$. This gives:

$$\begin{aligned}
\Delta Q_w &= MC_w(T_f - T_w) = 2MC_m(T_f - T_w) \\
\Delta Q_m &= MC_m(T_f - T_m) \\
\Delta Q_{drop} &= Mgh
\end{aligned}$$

Plugging these in to our initial statement and expanding, we have

$$2MC_mT_f - 2MC_mT_w + MC_mT_f - MC_mT_m = Mgh$$

We can cancel out the Ms, and divide by C_m to give

$$2T_f - 2T_w + T_f - T_m = \frac{gh}{C_m}$$

Rearranging, we get

$$T_f = \frac{1}{3}(2T_w + T_m + \frac{gh}{C_m}) \tag{1}$$

Whenever arriving at a pivotal point like this, it's good to take stock and make sure everything makes sense. First, the dimensions of $\frac{gh}{C_m}$ are $ms^{-2}m/(m^2s^{-2}K^{-1}) = K$, which agrees with the rest of the formula. Now, suppose you had initially but the Mgh term on the wrong side and wound up with a minus sign. How could we spot this error? Notice that if the metal starts at a higher point, more heat would be added, so the final temperature would be *greater*, indicating we need a plus sign.

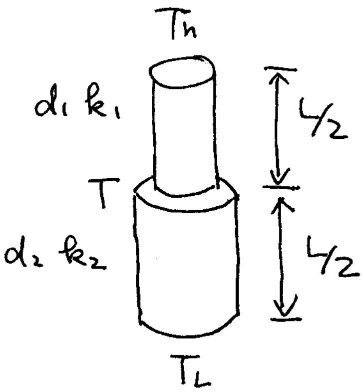
We are now ready to find the entropy change of this process. The first thing to note is that it is *irreversible*. Thus, we can't blindly apply our integral formula for entropy. Instead, we break our system up into two segments: the water and the metal. The temerature changes of each of these three components, then, can be modeled by reversible processes, since entropy is a state variable. The total entropy change would be the sum of these two [note that the effect of the heat from falling is automatically accounted for in using T_f and in our modeling]. We know that $\Delta Q = MC\Delta T$. This is easily converted to a differential relation: $dQ = MCdT$. Then, $dS = dQ/T = MC\frac{dT}{T}$. To find the entropy change, we just integrate:

$$\Delta S = MC\int_{T_i}^{T_f}\frac{dT}{T} = MC\ln\frac{T_f}{T_i}$$

We are bringing the metal from temperature T_m to T_f, so $\Delta S_m = MC_m \ln\frac{T_f}{T_m}$. Similarly, $\Delta S_w = 2MC_m\ln\frac{T_f}{T_w}$. Adding, combining and rearranging so that all of our terms are implicitly positive,

$$\Delta S = MC_m\big(2\ln\frac{T_f}{T_w} - \ln\frac{T_m}{T_f}\big) \tag{2}$$

4. (a) Heat flow through the two parts of the wire should be equal. If we say the temperature in the middle is T,



$$\dot{Q} = k_1 \pi \left(\frac{d_1}{2}\right)^2 \frac{T_h - T}{L/2}$$

$$= k_2 \pi \left(\frac{d_2}{2}\right)^2 \frac{T - T_L}{L/2} \quad \cdots ①$$

Solve for T.

$$k_1 d_1^2 (T_h - T) = k_2 d_2^2 (T - T_L)$$

$$(k_1 d_1^2 + k_2 d_2^2)\, T = k_1 d_1^2 T_h + k_2 d_2^2 T_L$$

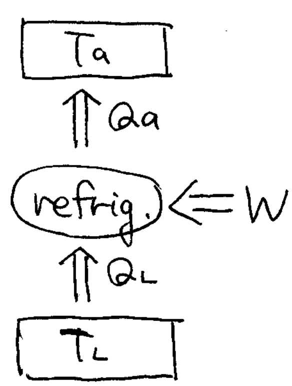
$$T = \frac{k_1 d_1^2 T_h + k_2 d_2^2 T_L}{k_1 d_1^2 + k_2 d_2^2} \quad \cdots ②$$

Put ② in ①;

$$\dot{Q} = k_2 \pi \frac{d_2^2}{4} \cdot \frac{2}{L} \left(\frac{k_1 d_1^2 T_h + k_2 d_2^2 T_L}{k_1 d_1^2 + k_2 d_2^2} - T_L \right)$$

$$= \frac{\pi}{2} \cdot \frac{1}{L} \frac{k_1 k_2 d_1^2 d_2^2}{k_1 d_1^2 + k_2 d_2^2} (T_h - T_L)$$

(b)



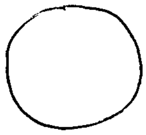
For a Carnot refrigerator,

$$CP = \frac{Q_L}{W} = \frac{T_L}{T_a - T_L}$$

$$\Rightarrow W = \frac{T_a - T_L}{T_L} Q_L$$

$$\text{power} = \frac{T_a - T_L}{T_L} \dot{Q}$$

PROBLEM 5



Mass M, density $\rho = \frac{M}{V}$,
specific heat C, emissivity e

The solid loses energy via Stefan-Boltzmann radiation:

$$\underset{\text{power radiated to environment}}{P} = -\underbrace{\frac{dQ}{dt}}_{\text{rate of heat flow into body}} = e\sigma A T^4$$

t for time

A the surface area

We know that the temperature change corresponding to a small heat flow into or out of the body is given by

$$dQ = MC\,dT \qquad \text{(definition of } C\text{)}$$

so
$$\frac{dQ}{dt} = MC\frac{dT}{dt}.$$

Equating these expressions for $\frac{dQ}{dt}$, $-MC\frac{dT}{dt} =$ [redacted] $e\sigma A T^4$

Separate variables: $-MCT^{-4}\,dT = e\sigma A\,dt$

<u>Integrate:</u>
$$-MC\int_{T_0}^{T_1} T^{-4}\,dT = e\sigma A\int_{t_0}^{t_1} dt$$

$$-MC\left[-\tfrac{1}{3}T^{-3}\right]_{T_0}^{T_1} = e\sigma A\,(t_1 - t_0)$$

$$\underline{\frac{MC}{3e\sigma A}\left[\frac{1}{T_1^3} - \frac{1}{T_0^3}\right] = \Delta t}$$

Note: $T_1 < T_0 \Rightarrow \Delta t > 0$

This is the answer; the rest is merely expressing it in terms of the given quantities.

$$M = \rho V = \rho \cdot \tfrac{4}{3}\pi r^3 \Rightarrow r^3 = \frac{3}{4\pi}\frac{M}{\rho} \Rightarrow r = \left(\frac{3}{4\pi}\frac{M}{\rho}\right)^{1/3}$$

$$\frac{M}{A} = \frac{\frac{4}{3}\pi r^3 \rho}{4\pi r^2} = \frac{\rho r}{3} = \frac{\rho}{3}\left(\frac{3}{4\pi}\frac{M}{\rho}\right)^{1/3}$$

So
$$\boxed{\Delta t = \frac{c}{3e\sigma}\cdot\frac{\rho}{3}\left(\frac{3}{4\pi}\frac{M}{\rho}\right)^{1/3}\left[\frac{1}{T_1^3} - \frac{1}{T_0^3}\right]}$$