PART A: Basics   (15 points)

1. **(1 point)** Suppose that u = [-3, 0, 0, 2, 5] and v = [-3, -2, 0, 2, 4]. What are the results of the following operations?

   (a)   u > 0   _____0 0 0 1 1_____

   (b)   v == u   _____1 0 1 1 0_____

2. **(1 point)** Consider the following MATLAB code that evaluates y for a predefined vector, x.

```
clear y
for i = 1:length(x)
   j = 0;
   if x(i) == 0 | x(i) < 0
      j = 1;
   end
   y(i) = j;
end
```

   Write a single MATLAB statement to accomplish the above.

   » __ y = (x < 0 | x == 0) + 0*(x > 0)_____

3. **(1 point)** When the following MATLAB program is executed what would be the output from the last statement?

```
p=0; q=0; r=0;
for i = 1 : 2 : 3
      p = p + 1;
      for j = 1 : i
         q = q + 1;
      end
end
disp([p,q])
```

   _2 4_____

4. **(1 point)** Given that `wt = [2, 5, 6, 8, 3];` what will be the value of `num` after MATLAB executes the following set of command lines?

```
num = 0;
for i = 1 : length(wt)
   if (wt(i) <= 5)
      num = num + 1;
   end
end
disp(num)
```

```
                    3
```

5. **(1 point)** Write a single MATLAB statement to accomplish exactly the same as the code in the above problem.

```
»   disp(sum(wt<=5))   or   disp(length(find(wt<=5)))
```

6. **(4 points)** If `p = 3.5`, `q = 2`, and `k = -5.5`, what will be the result of the following expressions?

(a) `(p + q) >= k`	1
(b) `(p < 5) & (k > 0)`	0
(c) `(p > q)	(p < q)`	1
(d) `p + (q >= k)`	4.5

7. **(4 points)** Given below are two incomplete `for` loops. Fill in the blanks of the MATLAB code such that the first loop is equivalent to the command `B = sum(A)`, and the second loop is equivalent to `C = sum(A')`, where A is a predefined (m x n) matrix.

```
[m, n] = size(A);
for j= __1____ : 1 : _n____
   B(j) = sum( ____A(:,j)____ )
end
for i= __1____ : 1 : ___m____
   C(i) = sum( ____A(i,:)____ )
end
```

8. **(2 points)** The height h of an object as a function of time t is described by the equation

$$h(t) = 20 + 10\,t - 2\,t^{2.3}$$

Solution to this equation for $h(t) = 0$ is known to exist in the neighborhood of $t = 3$. (Fill in the following blank)

```
h = @(t) 20+10.*t-2*t.^2.3
```

In one line, write a MATLAB instruction which uses the previous handle function to solve the equation $h(t) = 0$

```
>> _____fzero(h,3)_____
```

PART B: Functions (10 points)

1. **(6 points)** Fill in the blanks in the following MATLAB function `leap` which returns a `value` of 1 (true) or 0 (false) depending on whether the input argument `year` is a leap year or not. Recall that, by definition, a leap year is a multiple of 4 (but if it is a multiple of 100, it must also be a multiple of 400). Accordingly, note that the years 2000 and 1776 are leap but 1993 and 1900 are not. Hint: The MATLAB function `rem(m,n)` gives the remainder when m is divided by n.

```
function value = ____leap_____( ___year_____ )

if ___rem(year,4)==0_____
   value = 1;
    if rem(year,100)== 0
     if _____rem(year,400)==0_____
        value = ___1__;
     else
        value = __0___;
     end
    end
else
   value = __0___;
end
```

2. A solution is defined as *acid* when its pH is below 7, *neutral* when its pH is exactly 7, and *base* when its pH is above 7.

(a) (**2 points**) Write a MATLAB function called PH that receives a single pH value as input argument and returns a single string 'acid' or 'neutral' or 'base' as its output.

```
function s=PH(pH)
if (pH <7)
     s='acid';
elseif (pH == 7)
     s = 'neutral';
else
     s='base';
end
```

(b) (**2 points**) Suppose you are given a set of pH values val =[5.1, 7.0, 11, 12.5, 6]. Write a for loop to call the PH function to determine the corresponding acidity (i.e., acid, neutral or base).

```
for i=length(val)
  str{i}=PH(val(i));
end

or

val=[ 5.1, 7.0, 11, 12.5, 6 ];
for i=val
     fprintf('val=%4.1f is %4s\n',i,PH(i))
end
```

PART C: Linear Algebra   (12 points)

1. **(3 points)** Show that the following set of linear equations has no solution:

$$-4x + 5y = 10$$
$$12x - 15y = 8$$

a. What is the determinant of the coefficient matrix of these equations? (Fill the four blanks in the matrix A and the result)

$$A = \begin{pmatrix} -4 & 5 \\ 12 & -15 \end{pmatrix}, \ \det(A) = 0$$

b. What is rank(A)?

_____1_____

c. What is the rank of the augmented matrix? (Fill the six blanks in the matrix [A,b] and the result)

$$[A,b] = \begin{pmatrix} -4 & 5 & 10 \\ 12 & -15 & 8 \end{pmatrix}, \ \text{rank}[A,b] = 2$$

where b is the right-hand side vector.

2. **(6 points)** Answer the following questions:

(a) What MATLAB function calculates the determinant of a matrix A?

_____**det(A)**_____

(b) Does the matrix A above have to be square? (Circle the right answer)

$\boxed{\text{YES}}$ NO

(c) What does the backslash (\) operator do for an (n x n) invertible system of linear equations?

_____**provides the exact solution** _____

(d) What MATLAB function calculates the inverse of a matrix?

_____**inv**_____

(e) Does the matrix to be inverted have to be square? (Circle the right answer)

$\boxed{\text{YES}}$ NO

(f) Write a single MATLAB statement to solve the system of equations:

$$A x^2 = b$$

where A is a (n x n) square matrix of rank n, x and b are vectors of size $n \times 1$. A and b are given. In Matlab notation, $x^2 = x.*x$.

```
Find x: _____x=(A\b).^0.5_____or _____ x = sqrt(inv(A)*b)_____
```

3. **(3 points)** For what value of a will the following set have a solution in which both x and y are nonzero? Find the relationship between x and y.
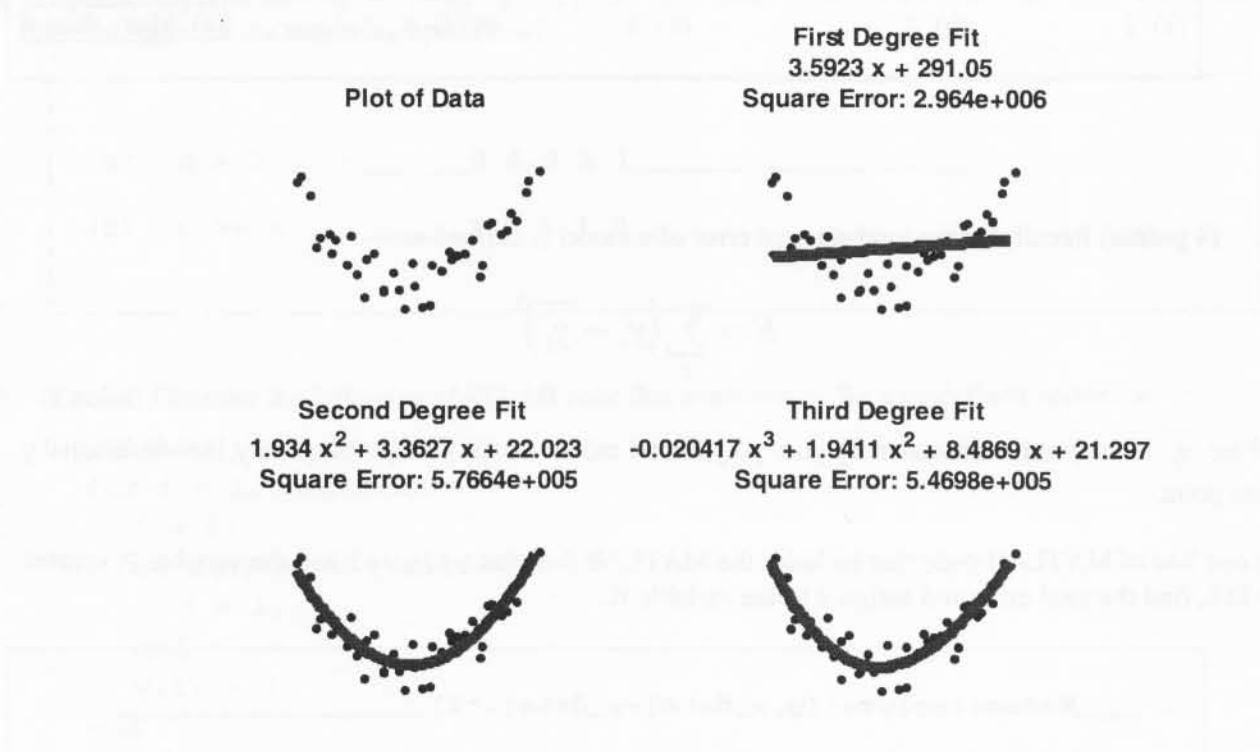
$$4x - ay = 0$$
$$-3x + 6y = 0$$

```
   a=8, x= any number ne 0, y=x/2
 or
   a=8, x=2y, y= any number ne 0
```

PART D: Least Squares Regression   (8 points)

Refer to the figure for the following question.

Plot of Data

First Degree Fit
3.5923 x + 291.05
Square Error: 2.964e+006

Second Degree Fit
$1.934 x^2 + 3.3627 x + 22.023$
Square Error: 5.7664e+005

Third Degree Fit
$-0.020417 x^3 + 1.9411 x^2 + 8.4869 x + 21.297$
Square Error: 5.4698e+005

1. **(1 points)** With the information above, the underlying phenomenon the data is extracted from is most closely approximated by (Circle the right answer):

(a)	A Linear Model	(b)	A Quadradic Model
(c)	**A Cubic Model**	(d)	A Non-Polynomial Model

2. **(2 points)** Assume you have variables in the MATLAB workspace called x_data and y_data which are arrays of data points taken from an experiment. You have reason to believe the data corresponds to a 4th order polynomial model. In one line, use the MATLAB function polyfit to find the parameters associated with your model and assign the parameters to a variable P.

```
>> _____P=polyfit(x_data,y_data,4)_____
```

3. **(1 point)** Assuming you have created the variable P in the previous question correctly, how many elements will the array, P, have? (Circle the right answer)

 (a) 1   (b) 2   (c) 3   (d) 4   **(e) More than 4**

4. **(4 points)** Recall that the total squared error of a model is defined as:

$$E = \sum_i \left(y_i - \overline{y_i} \right)^2$$

where $\overline{y_i}$ is the y value estimated by your polynomial model for the data point x_i and y_i is an individual y data point.

In one line of MATLAB code that includes the MATLAB function polyval and the variable P, created in D.1, find the total error and assign it to the variable E.

```
>> ____E=sum((polyval(p,x_data)-y_data).^2)_____
```