Print your name:
Signature:

SID
Discussion Section:

Math 54 Second Midterm Fall 2012 Instructor: D.-V. Voiculescu

This is a "closed book" exam, so you may not bring in or use notes or the textbook. Calculators are not allowed.

Please write your name , SID and Discussion Section # on everything you hand in, including this sheet of paper on which you have to provide the answer to Problem II (the true or false questions). For Problem I you must show the method and calculations you use to get the answers (write the solutions to the questions in Problem I in your blue book). The Requirement is 20 points.

Problem I (3+2+2+3+3 pts) Let A and B be the matrices: $A=\begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$ $B=\begin{pmatrix} -1 & 1 \\ 0 & 1 \\ -1 & 1 \\ 0 & 1 \end{pmatrix}$

a) Find the eigenvalues and eigenvectors of A.

b) Find an invertible matrix S and a diagonal matrix D so that $D = S^{-1} A S$.

c) Apply Gram-Schmidt to the columns of B to find an orthonormal basis of Col (B).

d) Find the 4x4 matrix of the orthogonal projection onto Col (B).

e) Find a least squares solution of $Bx = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

Problem II (7 pts, each question 1 pt). Check True or False .

	True	False
a) If a, b, c, d $\in \mathbb{R}$ then $\sqrt{a^2+4b^2}\sqrt{c^2+d^2} > \lvert ac+2bd \rvert$		
b) The inverse of an orthogonal matrix is an orthogonal matrix		
c) $\left\langle \begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} c \\ d \end{pmatrix} \right\rangle = \det \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ is an inner product on $\mathbb{R}^2$		
d) If E, F are symmetric 2x2 matrices, then so is EF.		
e) If W is a subspace of $\mathbb{R}^4$ and {x, y} and {z, t} are bases of W and $W^{\perp}$, then {x, y, z, t} is a basis of $\mathbb{R}^4$.		
f) An orthogonal 2x2 matrix is always diagonalizable		
g) If M is a 3x2 matrix then rank (A) + nullity (A^T) = 3		