Math 54 First Midterm Fall 2006 Instructor: D.V. Voiculescu

This is a "closed book" exam, so you may not bring in or use notes or the textbook. Calculators are not allowed.

Please write your name, SID and Discussion Section # on everything you hand in, including this sheet of paper on which you have to provide the answers to Problem III (the true or false questions). For Problems I and II you must show the method and calculations you use to get the answers (write the solutions to these in your blue book). The Requirementis 20 points.

Problem I (5pts). Solve by Gauss elimination the system :

$$x+y = 1,\ x+z = -1,\ t+y = 1,\ t+z-2w = -1$$

Problem II (4pts). Orthogonalize by Gram-Schmidt in $\mathbb{R}^4$ the vectors:

$$(1,1,0,0),\ (-1,0,1,0),\ (0,1,0,1)\ .$$

ProblemIII (11pts, each question 1 pt). Check True or False.

	True	False
a) $\{a+bX \mid (a,b)\in\mathbb{R}^2\}$ is a subspace of the vector space of polynomials of degree ≤ 3.	X	
b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ is a product of elementary matrices		X
c) $(1,1,1) \in \operatorname{span}\{(1,2,3),\ (2,1,1)\}$		X
d) $\cos^2 t$, $\sin^2 t$, $\cos 2t$ are linearly dependent in $C[0,1]$.	X	
e) In a vector space the intersection of 2 subspaces is always a subspace	X	
f) In a vector space the union of 2 subspaces is always a subspace.		X
g) if A is the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then A^{100} is invertible.	X	
h) the angle of the vectors $(1,1,1)$ and $(1,2,3)$ is 120° .		X
i) in $\mathbb{R}^3$, span $\{(1,2,3),\ (3,2,1)\}$ is a line.		X
j) the nullspace of $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 0 & \sqrt{3} & \sqrt{3} & 0 \end{bmatrix}$ contains a nonzero vector.	X	
k) $(1,1,0,0)$, $(0,1,1,0)$, $(0,0,1,1)$, $(1,0,0,1)$ is a basis in $\mathbb{R}^4$.		X

$\frac{5}{5}$

I. $\begin{cases} x+y=1 \\ x+z=-1 \\ t+y=1 \\ t+z-2w=-1 \end{cases}$

$$\begin{array}{c} \begin{array}{ccccc} x & y & t & z & w \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -2 & -1 \end{array}\right] \begin{array}{l} \\ +(-1)r_1 \\ \\ \\ \end{array} \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -2 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -2 & -1 \end{array}\right] \begin{array}{l} \\ \\ +r_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & -2 & -1 \end{array}\right] \begin{array}{l} \\ \\ \\ +(-1)r_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -2 & 0 \end{array}\right]$$

$\begin{cases} x+y=1 \\ -y+z=-2 \\ t+z=-1 \\ -2w=0 \end{cases}$

z is a free variable

let $z=a$, where a is real number.

$w=0$

$t=-1-z=-1-a$

$-y=-2-z$

$y=2+z=2+a$

$x+y=1$

$x=1-y=1-(2+a)$

$=-1-a$

Solution:

$x=-1-a$ where a is a real number

$y=2+a$

$t=-1-a$

$z=a$

$w=0$

4/4

II. $(1,1,0,0)=v_1$ $\quad$ Let $p_1 = v_1 = (1,1,0,0)$

$(-1,0,1,0)=v_2$ $\quad$ $p_2 = v_2 - \left(\text{proj } v_2 \text{ on } p_1\right) = v_2 - \frac{v_2 \cdot p_1}{p_1 \cdot p_1} p_1$

$(0,1,0,1)=v_3$

$$= (-1,0,1,0) - \frac{(-1,0,1,0)\cdot(1,1,0,0)}{(1,1,0,0)\cdot(1,1,0,0)}(1,1,0,0)$$

$$= (-1,0,1,0) - \frac{-1+0+0+0}{1+1+0+0}(1,1,0,0)$$

$$= (-1,0,1,0) - \frac{-1}{2}(1,1,0,0)$$

$$= (-1,0,1,0) + \left(\tfrac{1}{2},\tfrac{1}{2},0,0\right)$$

$$p_2 = \left(-\tfrac{1}{2},\tfrac{1}{2},1,0\right)$$

$p_3 = v_3 - (\text{proj. } v_3 \text{ on } p_2) - (\text{proj. } v_3 \text{ on } p_1)$

$$= (0,1,0,1) - \frac{(0,1,0,1)\cdot(-\frac{1}{2},\frac{1}{2},1,0)}{(-\frac{1}{2},\frac{1}{2},1,0)\cdot(-\frac{1}{2},\frac{1}{2},1,0)}\left(-\tfrac{1}{2},\tfrac{1}{2},1,0\right)$$

$$- \frac{(0,1,0,1)\cdot(1,1,0,0)}{(1,1,0,0)\cdot(1,1,0,0)}(1,1,0,0)$$

$$= (0,1,0,1) - \frac{0+\frac{1}{2}+0+0}{\frac{1}{4}+\frac{1}{4}+1+0}\left(-\tfrac{1}{2},\tfrac{1}{2},1,0\right) - \frac{0+1+0+0}{1+1+0+0}(1,1,0,0)$$

$$= (0,1,0,1) - \frac{\frac{1}{2}}{\frac{3}{2}}\left(-\tfrac{1}{2},\tfrac{1}{2},1,0\right) - \frac{1}{2}(1,1,0,0)$$

$$= (0,1,0,1) - \tfrac{1}{3}\left(-\tfrac{1}{2},\tfrac{1}{2},1,0\right) + \left(-\tfrac{1}{2},-\tfrac{1}{2},0,0\right)$$

$$= (0,1,0,1) + \left(\tfrac{1}{6},-\tfrac{1}{6},-\tfrac{1}{3},0\right) + \left(-\tfrac{1}{2},-\tfrac{1}{2},0,0\right)$$

$$= \left(-\tfrac{2}{6},\tfrac{2}{6},-\tfrac{1}{3},1\right)$$

$$p_3 = \left(-\tfrac{1}{3},\tfrac{1}{3},-\tfrac{1}{3},1\right)$$

So, $p_1 = (1,1,0,0)$

$p_2 = \left(-\tfrac{1}{2},\tfrac{1}{2},1,0\right)$

$p_3 = \left(-\tfrac{1}{3},\tfrac{1}{3},-\tfrac{1}{3},1\right)$