Math 54 First Midterm Fall 2006 Instructor: D.V. Voiculescu

This is a "closed book" exam, so you may not bring in or use notes or the textbook. Calculators are not allowed.

Please write your name, SID and Discussion Section # on everything you hand in, including this sheet of paper on which you have to provide the answers to Problem III (the true or false questions). For Problems I and II you must show the method and calculations you use to get the answers (write the solutions to these in your blue book). The Requirementis 20 points.

Problem I (5pts). Solve by Gauss elimination the system :

$$x+y = 1,\ x+z = -1,\ t+y = 1,\ t+z-2w = -1$$

Problem II (4pts). Orthogonalize by Gram-Schmidt in $\mathbb{R}^4$ the vectors:

(1,1,0,0), (-1,0,1,0), (0,1,0,1) .

ProblemIII (11pts, each question 1 pt). Check True or False.

	True	False
a) $\{a+bX \mid (a,b) \in \mathbb{R}^2\}$ is a subspace of the vector space of polynomials of degree ≤ 3.		
b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ is a product of elementary matrices		
c) $(1,1,1) \in \text{span}\{(1,2,3), (2,1,1)\}$		
d) $\cos^2 t$, $\sin^2 t$, $\cos 2t$ are linearly dependent in C[0,1].		
e) In a vector space the intersection of 2 subspaces is always a subspace		
f) In a vector space the union of 2 subspaces is always a subspace.		
g) if A is the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then A^{100} is invertible.		
h) the angle of the vectors (1,1,1) and (1,2,3) is 120° .		
i) in $\mathbb{R}^3$, span {(1,2,3), (3,2,1)} is a line.		
j) the nullspace of $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 0 & \sqrt{3} & \sqrt{3} & 0 \end{bmatrix}$ contains a nonzero vector.		
k) (1,1,0,0), (0,1,1,0), (0,0,1,1), (1,0,0,1) is a basis in $\mathbb{R}^4$.		