Math 54 - Midterm #2

August 2, 2004, 10:00am-12:00pm

Name:

This is a closed book, closed notes exam. Calculators are not allowed. You have two hours to complete the exam. To receive full credit, write legibly, show your work and write proofs in complete sentences. If you need more space, use the back of the page of the problem on which you are working.

Problem	Points	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
Total	120	

1. Let

$$A = \begin{bmatrix} 3 & -1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix}$$

(a) Find the eigenvalues of A.

(b) Find bases for the eigenspaces of A.

(c) Is A diagonalizable? If so, diagonalize A.

2. Let $W = \text{span}\{(1,-1,0,1),(2,1,0,0)\} \subseteq \mathbb{R}^4$.

(a) Find an orthonormal basis for W.

(b) Find the point $x \in W$ that is closest to $y = (2,-1,1,3)$.

3. Find the general solution of the ODE

$$f'' + 2f' + 6f = 0 \tag{1}$$

and find solutions f_1 and f_2 of (1) such that $f_1(0) = 1$, $f_1'(0) = -1$, $f_2(0) = 2$, and $f_2'(0) = 1$. Is $\{f_1, f_2\}$ a fundamental set of solutions for (1)?

4. For each of the following, determine if A is definitely diagonalizable, maybe or maybe not diagonalizable, or definitely not diagonalizable. Explain your answers.

(a) $A = \begin{bmatrix} -1 & -1 & 2 \\ 0 & -3 & -1 \\ 0 & 0 & 2 \end{bmatrix}$.

(b) $A = \begin{bmatrix} 4 & -1 & 2 \\ -1 & 1 & 6 \\ 2 & 6 & -2 \end{bmatrix}$.

(c) $A = B^T B$ for some $m \times n$ matrix B.

(d) $A = BC$, where B and C are square, diagonalizable matrices.

5. For each of the following, find (or show there does not exist) a square matrix B with real entries such that $B^4 = A$.

(a) $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$

6. Let A, B and S be $n \times n$ matrices with S invertible.

(a) Prove that A and $S^{-1}AS$ have the same eigenvalues.

(b) Prove that if A is invertible then AB and BA have the same eigenvalues.