Math 54 Professor K. A. Ribet

Midterm Exam April 3, 2007

This exam was an 80-minute exam. It began at 3:40PM. There were 4 problems, for which the point counts were 9, 6, 7 and 8. The maximum possible score was 30.

Please put away all books, calculators, and other portable electronic devices—anything with an ON/OFF switch. You may refer to a single 2-sided sheet of notes. When you answer questions, explain in words what you are doing: your paper is your ambassador when it is graded. Correct answers without appropriate supporting work will be regarded with great skepticism. Incorrect answers without appropriate supporting work will receive no partial credit. Please write your name on each page of this exam. At the conclusion, please hand in your paper to your GSI.

1. *Let* $A = \begin{bmatrix} 0 & 0 & 0 \\ -2 & -2 & -1 \\ 4 & 4 & 2 \end{bmatrix}$. *Calculate the characteristic polynomial of* A. *Determine bases for each of the eigenspaces of* A. *Decide whether or not the matrix* A *is diagonalizable.*

2. *Find numbers* x *and* y *so that the product* $\begin{bmatrix} -1 & 2 \\ -2 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ *is as close as possible to the vector* $\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix}$.

3. *Let* A *be a* 3×3 *matrix with eigenvalues 0, 1, 2 and corresponding eigenvectors* v_0, v_1 *and* v_2.

a. *Find bases for* $\mathrm{CS}(A)$ *and* $\mathrm{NS}(A)$.

b. *Find two different vectors x such that $Ax = 2v_1 + v_2$.*

c. *Describe the set of vectors x for which $Ax = v_0$.*

4. *Let A be an $n \times n$ matrix such that $A^2 = A$.*

a. *Show that the non-zero elements of the column space of A are eigenvectors for A and that the non-zero elements of the null space of A are eigenvectors for A.*

b. *Let $v_1, \ldots, v_r$ be a basis for the column space of A and let $w_1, \ldots, w_s$ be a basis for the null space of A. (For simplicity, imagine that both the column space and the null space are non-zero.) Explain very briefly why $r + s = n$.*

c. *Prove that the $r + s$ vectors $v_1, \ldots, v_r; w_1, \ldots, w_s$ are linearly independent.*