Math 54 Professor K. A. Ribet

Midterm Exam February 14, 2007

This exam was an 80-minute exam. It began at 3:40PM. There were 4 problems, for which the point counts were 7, 12, 7 and 4. The maximum possible score was 30.

> Please put away all books, calculators, and other portable electronic devices—anything with an ON/OFF switch. You may refer to a single 2-sided sheet of notes. When you answer questions, explain in words what you are doing: your paper is your ambassador when it is graded. Correct answers without appropriate supporting work will be regarded with great skepticism. Incorrect answers without appropriate supporting work will receive no partial credit. Please write your name on each page of this exam. At the conclusion, please hand in your paper to your GSI.

1. *Label the following statements as TRUE or FALSE, giving a short justification for your choice. There are six parts to this problem, two per page.*

a. *If the span of $v_1, \ldots, v_n$ contains w_1, w_2 and w_3, it contains the span of these three vectors.*

b. *If B is an $n \times 5$ matrix, the set of matrices $A \in M_{mn}$ such that $AB = 0$ is a subspace of M_{mn}.*

c. *If u is a vector and v is a non-zero vector, the projection of u on v is perpendicular to u.*

d. *If v and w are column vectors of length n and m, the set of $m \times n$ matrices A satisfying $Av = w$ either has no solutions or an infinite number of solutions.*

e. *If W and W' are subspaces of a vector space V, the set of vectors in V that belong to both W and W' is a subspace of V.*

f. *If v_1 and v_2 are non-zero vectors of $\mathbf{R}^3$ that form an angle of $120°$, then there is a unique scalar c so that v_1 and $v_2 - c \cdot v_1$ are perpendicular.*

2. *Use Gaussian elimination to determine whether or not the matrix* $\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$ *has an inverse. If it does, find the inverse.*

3. *Let A be the 4×4 matrix* $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 2 & -1 & 0 & 1 \\ 3 & 1 & 3 & 6 \\ 4 & 0 & 1 & 7 \end{bmatrix}$. *Find a basis for the null space of A. Also, find a basis for the subspace of $\mathbf{R}^4$ spanned by the columns of A.*

4. *Suppose that $x_1, \ldots, x_k$ are elements of $\mathbf{R}^n$ and that A is an $m \times n$ matrix. If the products $Ax_1, \ldots, Ax_k$ are linearly independent vectors in $\mathbf{R}^m$, show that the vectors $x_1, \ldots, x_k$ are linearly independent.*