Midterm 2

Name	
T.A	

The boxes below are for your scores, do not write in them! Write your solutions in the spaces provided after each problem.

1		10
2		15
3		15
4		15
5		10
Total		65

1. Find the numbers x and y which give the best solution (in the sense of least squares) to the inconsistent system of equations

$$\begin{aligned} 1 &= 2x + y \\ 3 &= y \\ 3 &= x + y \\ 5 &= x + y \end{aligned}$$

2. Let W be the set of all $(x, y, z, w) : x + y = z + w = 0$.

   (a) Find an orthogonal basis for W.

   (b) Find the orthogonal projection of $(2, 0, 3, 5)$ onto W.

   (c) Find the distance from $(2, 0, 3, 5)$ to W.

3. Let $A = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$.

(a) Find the eigenvalues of A.

(b) We know that there exist matrices S and T such that $A = STS^{-1}$, where T is upper triangular and S is invertible. Find T.

(c) Now find S.

4. Let $A = \begin{pmatrix} 6 & -1 \\ 4 & 2 \end{pmatrix}$.

(a) Find the eigenvalues of A.

(b) We know that there exist matrices S and T such that $A = STS^{-1}$, where T is upper triangular. Find T.

(c) Now find S.

5. For each of the following matrices A, you are asked to determine whether or not there exist matrices S and D such that $A = SDS^{-1}$, with S invertible and then with S orthogonal. In the appropriate place in the box below, write Y if S and D with the properties named exist, N if not, and M if there is not enough information to tell.

(a)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 3 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 3 & 3 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 3 & 3 \end{pmatrix}$$

(d)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

(e) A is a matrix whose characteristic polynomial is $X(X-1)(X-2)$.

	Invertible S	Orthogonal S
a		
b		
c		
d		
e		