Professor A. Ogus
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Midterm 1

Name	
T.A	

The boxes below are for your scores, do not write in them! Write your solutions in the spaces provided after each problem. Explain your reasoning in all cases: you may be graded on your explanations as well as on your answers.

1		20
2		20
3		15
4		15
Total		

1. Find X, if possible. If not, explain why not.

(a) $\begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} = X + \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} = X \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

(c) $\mathrm{X} = (1, 2, 3) \cdot (-1, 3, 4)$

(d) $X \begin{pmatrix} 1 & 3 & 1 & 4 \\ 2 & 6 & 4 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & 2 & 0 \end{pmatrix}$

2. Consider the matrices:

$$A := \begin{pmatrix} 1 & -1 & 1 & -2 & -3 \\ 1 & 0 & 1 & -1 & -4 \\ 2 & -2 & 2 & -3 & -5 \\ 3 & -2 & 3 & -4 & -9 \end{pmatrix} \qquad \tilde{A} := \begin{pmatrix} 1 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Assuming these are row equivalent:

(a) Find a basis for the row space of A from among the rows of $\tilde{A}$.

(b) Find a basis for the column space of A from among the colums of A.

(c) Find a basis for the null space of A.

(d) What is the rank of A?

3. In each of the following examples, you are given a sequence of vectors in a vector space V. Answer the question, explaining your answer clearly, using complete sentences. Full credit will not be given if you just answer yes or no.

   (a) Does the sequence $((2, 2, -1, 4), (1, 7, 3, 2), (1, 4, 3, -1))$, form a basis for the vector space $V = \mathbf{R}^4$?

   (b) Does the sequence $(x^2 - 2x + 2, x^2 + 2x, x^2 - 1, x^2 - 3x + 5)$ form a basis for the space of polynomials of degree less than or equal to 2?

   (c) Suppose that (v_1, v_2, v_3, v_4) spans the null space of a 5 by 7 matrix A of rank 4. Is the sequence (v_1, v_2, v_3, v_4) linearly independent?

4. Let $M_{2,2}$ denote the vector space of all 2×2 matrices, and if $A \in M_{2,2}$, let $tr(A)$ denote the sum of the diagonal terms, and let W be the set of all $A \in M_{2,2}$ such that $tr(A) = 0$.

(a) Find a basis for W.

(b) Compute the dimension of W.

(c) Find the coordinates of $\begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$ with respect to your basis.