Math 54

First midterm

Spring 2010

Instructor : John Lott

This is a closed everything exam. Please put away all books, calculators and other portable electronic devices.

You need to justify every one of your answers. Correct answers without appropriate supporting work will be treated with great skepticism (except where indicated for problem number 4). At the conclusion, hand in your exam to your GSI.

Write your name on this exam and on any additional sheets that you hand in. If you need additional paper, get it from me.

Problem	Score
1	
2	
3	
4	
5	
Total	

Your name ______________________________

Your GSI ______________________________

Discussion time ______________________________

Your SID ______________________________

1. For each of the following two matrices, find whether or not it is invertible.

If the matrix is invertible, compute the inverse. If it is not invertible, find a relevant property of the matrix (that one can see without doing any computations) and carefully explain why this property forces the matrix to be noninvertible.

a. (10 pts)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 4 \end{bmatrix}.$$

b. (10 pts)

$$B = \begin{bmatrix} 1 & 2 & 4 & 4 \\ 3 & 5 & -1 & -1 \\ 2 & 6 & 1 & 1 \\ 1 & 1 & 7 & 7 \end{bmatrix}.$$

2. Put

$$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

a. (10 pts) Find a basis for the column space of A from among the columns of A.
b. (10 pts) Find a basis for the null space of A.

3. (20 pts) Are the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$ linearly independent? Justify your answer.

4. True or false? If your answer is "true" then you should justify it. If your answer is "false" then you should provide a counterexample.

For each question, if you give the correct answer without providing a correct justification or counterexample then you will get one point. If you give the correct answer and also a completely correct justification or counterexample, then you will get two points.

a. If a vector $\mathbf{x}$ in $\mathbb{R}^2$ is not the zero vector then each of its entries is nonzero.

b. If A is a square matrix and the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution then the reduced row echelon form of A is the identity matrix.

c. In a linearly dependent collection of three vectors, each vector must be a linear combination of the other two.

d. The linear transformation associated to any 2×2 matrix is a rotation of $\mathbb{R}^2$.

e. If $AB = BA$ and A is invertible then $A^{-1}B = BA^{-1}$.

f. If $A^2 = I$ then A must be I or $-I$.

g. If A is a 3×3 matrix then $\det(5A) = 5\det(A)$.

h. If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution then so does the system $A\mathbf{x} = \mathbf{0}$.

i. If $AC = 0$ then $A = 0$ or $C = 0$.

j. If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ is a collection of vectors in a vector space V, and $\{\mathbf{v}_1, \ldots, \mathbf{v}_{p-1}\}$ spans V, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ spans V.

5. (20 pts) Suppose that $\mathbf{x}_1, \ldots, \mathbf{x}_k$ are vectors in $\mathbb{R}^n$ and that A is an $m \times n$ matrix. If the products $A\mathbf{x}_1, \ldots, A\mathbf{x}_k$ are linearly independent vectors in $\mathbb{R}^m$, show that the vectors $\mathbf{x}_1, \ldots, \mathbf{x}_k$ are linearly independent.