1. Let matrix B be defined by

$$B = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix},$$

and let $\mathbf{B}$ be a basis consisting of columns of B. Let $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ be a vector in $\mathbf{R}^2$. Find the $\mathbf{B}$-coordinates of x.

2. Let matrix A be defined by

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix}.$$

(a) Find all the eigenvalues of A.

(b) Diagonalize A if possible; otherwise show why A is not diagonalizable.

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3. Let A be an $n \times n$ matrix.

   (a) Let u be an eigenvector of A corresponding to an eigenvalue λ, and let H be the line in $\mathbf{R}^n$ through u and the origin. Explain why H is invariant under A in the sense that Ax is in H whenever x is in H.

   (b) Let K be a one-dimensional subspace of $\mathbf{R}^n$ that is invariant under A. Explain why K contains an eigenvector of A.

4. (a) Let subspace $\mathbf{W} = \mathbf{span}(u, v)$, where

$$u = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 10 \\ 1 \\ 3 \end{pmatrix}.$$

Find an orthonormal basis for $\mathbf{W}$ using Gram-Schmidt process.

(b) Let $A \in \mathbf{R}^{m \times n}$ be an $m \times n$ matrix and $b \in \mathbf{R}^m$ be an m-dimensional vector. Show that the normal equation

$$A^T A x = A^T b$$

has a solution for any such A and b.