Name and SID:

1. Solve linear systems of equations $A\,x = b$, where

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

using the row reduction algorithm.

2. Let

$$A = \begin{pmatrix} 1 & -1 & t \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

where t is a real parameter.

(a) For $t = 0$, find a basis of the column space and a basis of the null space of A.

(b) For $t \neq 0$, show that A is invertible.

3. Let $\mathbb{P}_2$ be the set of polynomials of degree at most 2, and define a map $\mathbf{T}$ from $\mathbb{P}_2$ to $\mathbb{R}$ as follows: let $u(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$ be a polynomial in $\mathbb{P}_2$. Then $\mathbf{T}\,(u(x)) = \alpha_0 + \alpha_1 + \alpha_2$.

   (a) Show that $\mathbf{T}$ is a linear transformation.

   (b) find the dimensions of the range space and the kernel of $\mathbf{T}$.

4. Let $\mathbf{V}$ be a vector space, and let $\mathbf{H}$ and $\mathbf{W}$ be two subspaces of $\mathbf{V}$. Define

$$\mathbf{S} = \{u + v | u \in \mathbf{H} \quad \text{and} \quad v \in \mathbf{W}.\}$$

Show that $\mathbf{S}$ is a subspace of $\mathbf{V}$.